Mathematics vis-à-vis Arithmetics

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Abstract. This study engages in the two terms arithmetics and mathematics which are frequently interpreted as having an identical meaning. Pupils and elementary school mathematics pre-service teachers use those terms as if they were the same. This study clarifies these terms and explores their definition among pre-service teachers as well as 6th and 7th grade pupils. Presentation of a historical sequence focuses on the development of and relation between arithmetics and the various areas of mathematics. Later on, we elaborate the terms arithmetics and mathematics as they are meant to be understood by and explained to the learners and present the way they are perceived by pupils and pre-service teachers. The research findings illustrate the confusion these terms create.

Keywords: arithmetics; mathematics; pre-service teachers.

Introduction

The relation between arithmetics and mathematics is described by the analogy to a language, namely the relation between spelling and writing (Peterson, 2001). Arithmetics is the foundation stone of mathematics as letters are the foundation stone of writing. Moreover, mathematics is presented as the queen of sciences and as such, when the fundamental laws of arithmetics are insufficient, learners can solve more complex questions by means of the other areas of mathematics (Stevens, 2011; Turgeman, 2006; Weintraub, 2004).

One of the frequent questions of learners, whether at elementary school or junior high school, is: "Teacher, which books should I bring tomorrow? Arithmetics or mathematics?" Elementary school pupils or their teachers say that they are studying arithmetics and sometimes they say they are learning mathematics. Moreover, the class board in elementary school displays the captions 'mathematics' and 'geometry' as two different subjects, geometry being one of the areas of mathematics.

The study examines those two terms and how it perceived by pupils from the 6th and 7th grade and by students of elementary school mathematics teaching.
Theoretical Background

The Israeli Managing Director Circular (Ministry of Education, Culture and Sport, 2006) which specifies the mathematics curriculum for elementary schools, is grounded in the perception that pupils should acquire and develop a numerical and geometric insight. This insight is based on the acquisition of terms and structures in arithmetics and geometry as a continuing process. The arithmetic insight can be achieved by mastering mathematical competences and by being familiar with fundamental facts while emphasising oral computations. The same applies to the relations between different arithmetic terms and relying on them when choosing strategies for solving questions and checking the answers. All this is done by using the mathematical language correctly.

In junior high school, the academic year in the 7th grade starts with revision of the laws of arithmetics familiar to the pupils from elementary school. Later on, the pupils learn algebraic expressions which comprise letters and numbers, emphasising the performance of arithmetic operations, acquaintance of mathematical terms and mathematical procedures. The concepts arithmetics and mathematics are described in literature as follows:

Mathematics: a language dealing with numbers and shapes, their properties and the investigation of their interrelations, while using signs and symbols. Mathematics consists of arithmetics, algebra, differential and integral arithmetics, geometry, trigonometry and more (Hebrew Encyclopaedia, 1972; Peterson, 2001; Stevens, 2011; Weintraub, 2004).

Arithmetics: an area of mathematics based on the four basic mathematical operations: addition, subtraction, multiplication, division, root extraction as well as on the order of operations between them. This area constitutes a fundamental part of mathematics studies and is essential for the understanding thereof (Avnion, 1997; Hebrew Encyclopaedia, 1953; Stevens, 2011; Wikipedia, 2014).

In the introduction to his book, Gazit (2004, p. 5) wrote:

… true, many people use the principles of arithmetics – the first and basic mathematical area… However, mathematics has additional areas, the most familiar among them are: algebra, plane geometry and solid geometry, analytical geometry, trigonometry, differential and integral arithmetics, statistics and probability.

To emphasize the relations of arithmetics and all the other areas of mathematics, a Graphic Mathematics Model is drawn (Figure 1 - below). The Model illustrates some of the areas of mathematics.

The relations between arithmetics and the other branches - areas of mathematics are exemplified by bi-directional arrows. Those arrows show the reciprocal connections of arithmetics and all the other areas of mathematics.
The graphic Mathematics Model (Fig. 1) and the following chronological review indicate the development of the areas of mathematics, in general, and of arithmetics, in particular, and how they are intertwined.
Historical-Chronological Review

Researchers concur to some extent that mathematics (counting methods, four basic mathematical operations and geometric computations: arithmetics and geometry) was developed in Egypt at about 4000 B.C. (Gazit, 2004). Nevertheless, there are evidences related to the need for counting which dates back to 25,000 B.C. (Arbel, 2005). The Greek mathematicians distinguished between the science of numbers (arithmetics – number in Greek arithmos and in Latin arithmetica) and the wisdom of computation (logistics) (Dagon, 1955; Smith, 1958). Pythagoras (6th century B.C.) and his disciples worshiped whole numbers (Arbel, 2005), arguing that everything is a number, everything is arithmos. This view was proven as incorrect when they calculated the length of a diagonal of a square whose side was 1 length unit. As a result the Pythagorean sect disbanded. Later, the mathematicians embraced the geometric approach which facilitated proofs without reference to numbers (Unguru, 1989a). Archytas (5th-6th centuries B.C.), one of Pythagoras disciples, divided the engagement in mathematics into four areas (a division maintained for about 2000 years). Absolute numbers – arithmetics, useful numbers – music, sizes in position – geometry, sizes in motion – astronomy. The Pythagoreans raised the number to a level of religion, a religion of numbers (Arbel, 2005; Dagon, 1995; Gazit, 2004).

Euclid (3rd century B.C.) collected, edited and enhanced the geometric material written by his predecessors. Until our present days, the Euclidean geometry is named after him. Archimedes (3rd century B.C.) conceived the fundamentals of integral arithmetics (Arbel, 2005). Claudius Ptolemy (2nd century A.C.) made a great contribution to trigonometry and Abu al-Wafa’ (10th century A.C.) continued to develop it. In the year 90 A.C., Nicomachus of Gerasa wrote "Introduction to Arithmetics" – the first work separating arithmetics from geometry.

During the Middle Ages, the term arithmetics was not common, perhaps because it did not have a Latin origin. People were accustomed to call it 'Greek arithmetics' (or in Latin 'numerorum scientica'). In 1116 A.C., arithmetics was referred to as 'arismetricis', in 1140 A.C. it was called 'arismetrica' and 50 years later Fibonacci used the word 'rismetrica' (Smith, 1958).

Viete (1591) sets clear boundaries between the logistica numerosa – arithmetics – and logistica speciose – algebra. According to him, arithmetics deals with specific numbers and the regular arithmetic operations with them whereas algebra constitutes a method of acting with the numerical structures connecting between occurrences (Unguru, 1989b). The two numerical areas of study continued to be studied separately until the invention of the printing press.

Then, the more aristocratic name 'arithmetics' combined the two disciplines. This term is not universal. Until today, the Germans refer to arithmetics as to the theoretical part and use the word Rechnen [calculate] for the ancient logistics and the French call is calcul [calculation] (Smith, 1958).

Pierre de Fermat (17th century) developed a theory which comprised much of what is called at present analytical geometry and at the same period Rene Descartes actualised and in fact invents the concept of the analytical geometry (Arbel, 2005). Moreover, Descartes was the first to present the infinitesimal
computation which constituted the basis of differential and integral arithmetics developed by Leibnitz and Newton after Descartes' death. In 1664, Isaac Barrow presented in his lectures materials which were the origins of differential arithmetics. In 1675, Isaac Newton and Gottfried Wilhelm Leibniz developed the differential and integral arithmetics. In 1690, Jacques Bernoulli used for the first time the term integral. In 1920, Hardy Godfrey Harold solved the Waring problem by a method which integrated analysis with arithmetics. Its importance resides in the fact that it is also applicable in the case of very difficult arithmetic problems.

The historical review above reinforces the fact that arithmetics is an initial basis and inseparable part of all areas - branches of mathematics.

Research Statement

Hashiv (anon) and Peterson (2001) argue that most pupils do not know to define mathematics or what is the difference between mathematics and arithmetics. Kyriakides, Meletiou-Mavrotheris and Prodromou (2016) state that pupils have fundamentally narrow viewpoint of mathematics as being primarily computation and arithmetics. Latterell and Wilson (2016) state that elementary teachers and elementary students tend to describe mathematics as arithmetics operations and computations.

Aharoni (2011) indicates that at elementary school arithmetics teaching is actually elementary mathematics in which basic topics are studied. For example: essence of the number as well as meaning of the basic mathematical operations, derived from the rules and order of the mathematical operations. Mathematics is unique in that it simplifies the most basic thinking processes.

What distinguishes the math is that it abstracts the basic thinking processes. We have examined the above with pre-service teachers and pupils.

Research aim: explore how the terms mathematics and arithmetic are perceived by 6th graders, 7th graders and pre-service teachers.

Materials and Methods

Research Population

76 pupils from the 6th grade.
65 pupils from the 7th grade.
56 pre-service teachers [hereafter - "students"] in their 4th year of studies, learning to become elementary school mathematics teachers.

Research Instrument

An open-ended questionnaire (see Appendix A) comprising three items. These items aimed to check the meaning attributed by the participants (a subjective interpretation) to the terms arithmetics and mathematics.
Results
The table in Appendix A presents the questionnaires results. The participants' answers were divided into categories. The name of which was determined according to the answers. The data were quantitatively analyzed while finding relations between the results.

Item No. 1: What is Arithmetics?
Most of the students (89%) knew to explain what is arithmetics. Conversely, only about half of the 6th and 7th graders - 47% of the 6th graders and 62% of the 7th graders - could explain the essence of this term.

Examples of answers to the item 'what is arithmetics': "a learning subject at school, which helps us to advance in life", "arithmetics is mathematics for elementary school", "a simple computation of numbers, i.e. plus, minus, division, multiplication".

Item No. 2: What is Mathematics?
79% of the students knew to explain this term. 31% of the 7th graders (less than half the percentage of the students) and 18% of the 6th graders (about one quarter of the percentage of the students) could explain the essence of mathematics.
Examples of answers to the item 'what is mathematics': "an alternative and more difficult word for arithmetics", "mathematics is advanced arithmetics", "mathematics is the generalization. It is arithmetics, geometry, gematria\(^1\)"

**Item No. 3: In Your Opinion, What is the Relation between Arithmetics and Mathematics?**

Three answers were obtained for this item. 79% of the students responded there was a relation and specified what was its essence. So did 35% of the 7\(^{th}\) graders (about half of the percentage of the students). On the other hand, only 5% of the 6\(^{th}\) graders explained this relation.

92% of the 6\(^{th}\) graders believed that there was no difference between arithmetics and mathematics. 62% of the 7\(^{th}\) graders and 14% of the students thought so too. 3% of the 6\(^{th}\) and 7\(^{th}\) graders as well as 7% of the students gave no answer to this item.

\(^1\) Gematria originated as an Assyro-Babylonian-Greek system of alphanumeric code/cipher later adopted into Jewish culture that assigns numerical value to a word/name/phrase. It is also used in Greek and Arabic.
Table 3: Results (%) for the relation between arithmetics and mathematics

Examples of answers to the item ‘what is the relation between arithmetics and mathematics’: "There is no relation", "a similar subject but mathematics is more difficult. Arithmetics is for elementary school and mathematics is for post-elementary school", "in my opinion arithmetics is for children and mathematics for adolescents", "arithmetics is the basis of mathematics, we learn first the basic material and from there we shift to mathematics".

Although they had not been asked, the participants related in their answers to the degree of difficulties they encountered in arithmetics and mathematics. Below are their answers and the analysis thereof:

39% of the 6th graders maintained that arithmetics was an easy subject whereas mathematics was a difficult one. Compared to them, 5% of the 7th graders and 4% of the students thought like them and chose to indicate it in their answers.
Table 4: Results (%) for the degree of difficulties encountered in arithmetics

Examples of answers to the item 'degree of difficulties encountered in arithmetics': "Arithmetics is the easier level for beginning children (addition and subtraction, comparison between integers)"

Table 5: Results (%) for the degree of difficulties encountered in mathematics

Examples of answers to the item 'degree of difficulties encountered in mathematics': "... much more difficult than arithmetics".

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Summary and Conclusions

This study aimed to explore how the terms mathematics and arithmetics and the relation between them are perceived by pupils in the 6th and 7th grade and by students of elementary school mathematics teaching.

The results illustrated that less than half the 6th graders could explain what is arithmetics and less than one-fifth were able to explain what is mathematics. This accounted for the finding that pupils of the 6th grade (92%) responded there was no relation between arithmetics and mathematics. It is noteworthy that the pupils have been learning arithmetics for six years and still found it difficult to clarify this term. About 40% of the 6th graders related to the degree of difficulties they had in arithmetics and/or mathematics. Underscoring the 'easy' arithmetics versus the 'difficult' mathematics was in line with the other results discussed above.

About 60% of the 7th graders could define arithmetics and some 30% knew to define mathematics. Two-thirds of the 7th grade pupils thought there was no relation between mathematics and arithmetics and one-third believed there was a relation between them. We realized that there was a change in the reference to and understanding of the relation between mathematics and arithmetics among pupils moving from the 6th to the 7th grade. 92% of the 6th graders claimed there was no relation between mathematics and arithmetics as compared to 62% of the 7th graders. It was obvious that in spite of the gap of one year, the transition to junior high school affected the differentiation in the answers. Nevertheless, most of the 7th graders, whose learning subject was already called mathematics, did not know to explained what was the essence of mathematics and did not think there was a relation between mathematics and arithmetics.

The students showed no meaningful difference between the percentage of answers to each of the questions. The results indicated that the students had a more consolidated opinion about the essence of arithmetics, the essence of mathematics and the relation between them. About 10% of the students failed to explain the essence of arithmetics and approximately 20% did not know to explain the essence of mathematics and the relation between the two.

The pupils’ answers indicated that there was no distinction between arithmetics and mathematics. One learns arithmetics from the 1st until the 6th grade and mathematics from the 7th grade and above. Some think that arithmetics is the order of operations in exercises, such as: multiplication, division, addition and subtraction while in mathematics pupils learn equations, algebra and so on. For certain learners of mathematics implies exercises with fractions ('complicated exercises') (see Appendix C, tables 1-5).

The historical review described above and the research results emphasise that arithmetics is the theory of numbers or to be more precise the theory of operations with numbers. In order to increase pupils and their teachers' awareness of an appropriate and accurate use of these terms, we described the relation between arithmetics and mathematics by means of the Mathematics Model. The Model shows arithmetics at the vertex of a pyramid which feeds
(mutual feedback) all the other divisions/areas of mathematics located at the base of the pyramid.

Learners use these terms currently but with no accuracy and conceptualisation. The daily use of the terms mathematics and arithmetics, as if they were one term, while not paying attention to the relation and/or the difference between them, is one of the factors affecting the lack of clarity and distinction between these two terms.

We should relate to the fact that the subject of arithmetics learnt in elementary school is an incomparably crucial and essential milestone. Arithmetics forms an important and inseparable part of mathematics studies in elementary school, junior high school and above. Although the arithmetic operations are apparently a simple procedure, elementary school pupils should understand and correctly use the order of the basic mathematical operations and the process of computation. This helps for example to prevent difficulties in understanding the mathematical operations and connections in algebraic expressions which is the first topic the 7th graders encounter when starting the junior high school.

Mathematics studies in the wider sense and not necessarily only arithmetics studies constitute the basis for teaching organized rational thinking. Arithmetics studied at elementary school is an important and inseparable part of mathematics studied in elementary school and later on. Arithmetics, as the basis of mathematics, is an extensive area with quite a few nuances. Mathematics teachers in general and students in particular who understand this, are endowed with the orientation for proper teaching.

From the very start, when pupils comprehend the difference between the terms 'arithmetics' and 'mathematics' and express themselves correctly, it implies that they understand the meaning of each word. This is the opening to the continued appropriate mathematical conduct, grounded in the knowledge of definitions and understanding of processes.

We want to end with a short story taken from the field:

During one of the mathematics lessons in the 7th grade where we are teaching, a teacher (from a difference discipline) came into the classroom to make an announcement. He saw exercises written on the class board, turned to the pupils and asked: "When do we move from arithmetics to mathematics?" When asked: "Why did you ask this question?" he replied: "Because arithmetics is for beginners and mathematics for higher grades. It is all the same thing".
References


STIPS site (2011). *What is the difference between mathematics and arithmetics?*. [http://www.stips.co.il/ask/224764/%D7%9E%D7%94-%D7%99%D7%9A%D7%94-%D7%9E%D7%94%D7%91%D7%93%D7%9C-%D7%91%D7%99%D7%9E-%D7%9A%D7%95-%D7%9C%D7%97%D7%A9%D7%91%](http://www.stips.co.il/ask/224764/%D7%9E%D7%94-%D7%99%D7%9A%D7%94-%D7%9E%D7%94%D7%91%D7%93%D7%9C-%D7%91%D7%99%D7%9E-%D7%9A%D7%95-%D7%9C%D7%97%D7%A9%D7%91%). Accessed 10.1.2015. [Hebrew]


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Appendix A
The questionnaire form

Name: ____________________  Age: _________

What is arithmetics?
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________

What is mathematics?
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________

In your opinion, what is the relation between arithmetics and mathematics?
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
## Appendix B

### Results of the questionnaire

<table>
<thead>
<tr>
<th>% of students</th>
<th>% of 7th graders</th>
<th>% of 6th graders</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>89</td>
<td>62</td>
<td>47</td>
<td>Comprehensive explanation</td>
</tr>
<tr>
<td>11</td>
<td>38</td>
<td>53</td>
<td>Irrelevant/incorrect explanation</td>
</tr>
<tr>
<td>79</td>
<td>31</td>
<td>18</td>
<td>Comprehensive explanation</td>
</tr>
<tr>
<td>21</td>
<td>69</td>
<td>82</td>
<td>Irrelevant/incorrect explanation</td>
</tr>
<tr>
<td>79</td>
<td>35</td>
<td>5</td>
<td>There is a relation</td>
</tr>
<tr>
<td>14</td>
<td>62</td>
<td>92</td>
<td>There is no difference between them</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>3</td>
<td>Did not respond</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>39</td>
<td>Believe that arithmetics is an easy subject</td>
</tr>
<tr>
<td>96</td>
<td>95</td>
<td>61</td>
<td>Did not refer to the level of difficulty</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>39</td>
<td>Believe that mathematics is a difficult subject</td>
</tr>
<tr>
<td>96</td>
<td>95</td>
<td>61</td>
<td>Did not refer to the level of difficulty</td>
</tr>
</tbody>
</table>
Appendix C
Examples of additional answers to the questionnaire

<table>
<thead>
<tr>
<th>Item No. 3: What in your opinion is the relation between arithmetics and mathematics?</th>
<th>Item 2: What is mathematics?</th>
<th>Item 1: What is arithmetics?</th>
<th>Age</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pupils of the 6th grade</td>
<td>The relation is that both values have the same meaning</td>
<td>Mathematics = a synonym of arithmetics. The basic terms of arithmetics are called mathematics</td>
<td>Arithmetics = a subject studied in elementary, junior high and high school and we engage in it day-by-day (at school, restaurant, home…)</td>
<td>11+</td>
</tr>
<tr>
<td></td>
<td>The relation is that in both we use numbers and mathematical operations</td>
<td>Mathematic is multiplying or dividing big numbers</td>
<td>Arithmetics is adding integers and easy numbers</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>The relation between arithmetics and mathematics is the fact that they &quot;are connected&quot; to exercises and comparisons between all the areas</td>
<td>Mathematics is exercises on a high level with fractions, decimal numbers and comparison on a high level</td>
<td>Arithmetics implies exercises and comparisons of small numbers: division, multiplication, addition and subtraction</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>The relation is that both are subjects which deal with numbers</td>
<td>Mathematics is for learners in higher grades, from the 5th grade and above</td>
<td>Arithmetics is for younger children in the 1st, 2nd and 3rd grades</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>The relation between them is in fact a relation of similarity, they are very similar. The basis of both is the basis of almost</td>
<td>Mathematics is a general term which is divided into two parts: arithmetics and geometry</td>
<td>Arithmetics is a kind of mathematics which constitutes half of the wide term called mathematics</td>
<td>11</td>
</tr>
</tbody>
</table>
### Pupils of the 7th grade

<table>
<thead>
<tr>
<th>The relation is that mathematics is a small part of algebra which is studied in junior high school</th>
<th>Arithmetics is the basis of mathematics and we start learning it in elementary school</th>
<th>12.5</th>
<th>Omri</th>
</tr>
</thead>
<tbody>
<tr>
<td>The relation is that both are similar subjects with calculations and exercises</td>
<td>Exercises of addition, subtraction, multiplication and division for the higher grades, 4th-12th grades</td>
<td>Exercises of addition, subtraction, multiplication and division for the 1st-3rd grades</td>
<td>12</td>
</tr>
<tr>
<td>The relation is that mathematics is the generalisation. It is arithmetics, geometry</td>
<td>Arithmetics is a body within mathematics. It is a sub-subject within the wider subject (mathematics)</td>
<td>12</td>
<td>Lior</td>
</tr>
<tr>
<td>The relation between arithmetics and mathematics is that arithmetics is included within the subject of mathematics</td>
<td>Mathematics is a subject which encompasses all the types of arithmetics, computations with equations and so on</td>
<td>Arithmetics is thinking about calculation of numbers, their totals, quotient, product, etc.</td>
<td>12</td>
</tr>
<tr>
<td>---------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td>----</td>
</tr>
<tr>
<td>The relation between the two is that arithmetics is included in the term mathematics and both are in fact a kind of calculations</td>
<td>Mathematics is a term which comprises algebra, geometry, gematria, etc.</td>
<td>Arithmetics is in fact computation – exercises and so on</td>
<td>12.5</td>
</tr>
<tr>
<td>The relation is that both are a similar subject but mathematics is more difficult. Arithmetics is for elementary school and mathematics is for post-elementary school</td>
<td>A subject studied at school which is necessary for the future</td>
<td>A subject at school facilitating our progress in life</td>
<td>12.8</td>
</tr>
</tbody>
</table>

**Students**

<table>
<thead>
<tr>
<th>The relation is that we need arithmetics in order to reach mathematics</th>
<th>Mathematics is a proof of arithmetics</th>
<th>Arithmetics is computation I think</th>
<th>25</th>
<th>Dotan</th>
</tr>
</thead>
<tbody>
<tr>
<td>The relation is that arithmetics is part of mathematics</td>
<td>A very wide area dealing with numbers, algebra, geometry and others</td>
<td>A subject/area which is derived from mathematics and constitutes its basis</td>
<td>24</td>
<td>Gal</td>
</tr>
<tr>
<td>The relation between them is that mathematics consists of arithmetics. Without arithmetics we cannot develop in mathematics</td>
<td>Mathematics focuses on quantities, spaces, structures and changes. Its development took hundreds of years and it continues to do so. It is learnt at post-elementary</td>
<td>Arithmetics is the four basic mathematical operations studied at elementary school – addition, subtraction, multiplication and division. Each operation is calculation of at</td>
<td>44</td>
<td>Noa</td>
</tr>
<tr>
<td>schools and higher education institutions</td>
<td>least two numbers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>------------------</td>
<td>------------------</td>
<td>------------------</td>
<td></td>
</tr>
<tr>
<td>The relation is that mathematics is based on the foundations of arithmetics</td>
<td>Proofs, theorems which connect the numbers and are connected by them</td>
<td>The basic operations on which mathematics is based: addition, subtraction, multiplication, division</td>
<td>23</td>
<td>Nofar</td>
</tr>
</tbody>
</table>