Effects of Warm-Up Testing on Student Learning

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Abstract. The assessment of student preparedness for entry-level university calculus has been of interest in recent years. Many institutions have adopted diagnostic tests as a means to assess foundational skills. We introduce a new testing-style, the Warm-Up Test, which occurs very early in the semester and only tests concepts from prerequisite courses that will be used to develop the new concepts in the course to come. Despite the large size of the course, Warm-Up Tests are not of a multiple choice format in order that rich feedback may be given by graders. Warm-Up Tests may also make up part of a student's grade, shifting weight from a high-stakes final exam. We analyze the predictive ability of this new form of assessment upon student performance on later assessments throughout the course, and we discuss this analysis as well as potential biases and possible future avenues of research.

Keywords: assessment; diagnostic testing; undergraduate mathematics; warm-up test.

1. Introduction

Recent curriculum changes at the secondary level in Ontario have resulted in a challenging environment for teaching and learning at the post-secondary level. The addition, removal, or change in the way that various mathematics concepts are introduced and reinforced in high schools have resulted in a difference in the alignment of these foundational curricula. This is not a new phenomenon; dealing with curriculum and curriculum-delivery changes has been an ongoing challenge for decades (Cooney, Bell, & Fisher-Cauble, Sanchez, 1996). There has been much interest in ensuring that students entering university continue to be as well-prepared as possible for their first university mathematics courses.

One strategy that has been well-explored in the literature is that of a diagnostic test. A number of studies have been conducted in the area of diagnostic testing, outlining and evaluating the structure, rationale, and efficacy of diagnostic tests administered upon enrolment in a post-secondary mathematics course. For example, online learning resources have been used for this purpose (Beevers,
Bishop, & Quinney, 1998), but these resources were not mandatory and may not have been utilized fully by all students. A novel approach has been utilized involving paired questions in an attempt to measure the sorts of questions that students may be liable to make a slip on despite having a solid understanding of the material (Lee & Robinson, 2005). Another approach has utilized a mandatory diagnostic test (Carr et al. 2013), requiring a 90 percent score to pass but allowing multiple attempts. A variety of styles of diagnostic tests have been used, each perhaps with its own benefits or specific purposes. A recent study showed that students largely believed diagnostic testing to be a positive and beneficial idea, but students stressed the need for improved communication (Ní Fhloinn, Macan Bhaird & Nolan, 2014). Examining the literature, however, reveals that multiple choice questions are almost always used exclusively within diagnostic tests and so written feedback to students is necessarily limited or altogether absent. This is significant, as research suggests that feedback may have a powerful influence upon student learning as well as having indirect effects such as an increase in the development of interest via a variety of means (Hattie & Timperley, 2007; Rakoczy et al., 2013). Kearney et al. (2013) agreed that student engagement is increased by the presence of feedback and pointed out that prior research shows "students who are more fully engaged in their own learning perform better academically than their non-engaged peers." (Kerney et al., 2013)

Motivated and influenced by these previous findings, our aim is to introduce a new form of assessment, called a "Warm-Up Test," that captures the spirit of a diagnostic test, but with a few important differences. Our Warm-Up Test occurs within the first two weeks of class and constitutes part of a student's overall grade. Warm-Up Tests are structured identically to "actual" term tests and do not contain multiple choice questions. The content appearing on the test is not limited to basic arithmetic and simple mathematics, but includes ideas from prerequisite courses that will squarely be used later within the course to build or prove new concepts; this strategy is one that has been explored in a study finding that interim testing of prior material facilitates learning of subsequent new material (Wissman, Rawson & Pyc, 2011). Finally, the Warm-Up Tests are graded quickly and returned to students with detailed, written, and personalized feedback so that they have the opportunity to fill in any perceived holes in their background knowledge so that they might be prepared for the rest of the course. We wish to analyze whether changing the structure of our first-year calculus course to include this type of assessment has an impact upon student learning and/or performance throughout the rest of the course.

This paper is laid out as follows: In Section 2, we discuss the history and background of first-year calculus at the University of Guelph. In Section 3, we discuss in greater detail the differences between a Warm-Up Test and a typical diagnostic test. In Section 4, we outline our study, summarizing some of our interesting results in Section 5. We provide a brief discussion of these results and subsequently potential biases in Sections 6 and 7. Final future endeavours as a result of this work are discussed in Section 8.
2. History and Background: First-Year Calculus at the University of Guelph

There are currently three dedicated introductory calculus courses offered at the University of Guelph: Math*1030 (Business Mathematics), taken chiefly by students pursuing Business or Economics degrees; Math*1080 (Elements of Calculus I), taken mainly by students enrolled in the Biological Sciences; and Math*1200 (Calculus I), typically taken by Physical Science and Engineering students. The latter of these three, Math*1200, is the course that will be discussed exclusively for the remainder of this paper.

Calculus I is a course that has experienced astonishing levels of growth, like many entry-level calculus courses in recent times; this is mainly due to recent expansion and vastly-increased student intake in the School of Engineering. For example, in the Fall 2009 semester, 537 students were enrolled at the end of the course, compared with 726 at the end of the Fall 2014 semester. This represents an increase of over 35 percent in just five years. Support from the University of Guelph for Teaching Assistantships has stayed roughly in-line with growth, and as of Fall 2014, 420 hours of Teaching Assistant support was granted over the semester. This is enough to allow for tests and the final exam to continue to be hand-graded with written feedback, despite these large numbers. Multiple choice is not used as a method for assessment in this course. Students taking this particular course are in degree programs that utilize mathematical skills such as those learned in Calculus I frequently throughout their degrees. As such, despite growing numbers, the feedback and learning that is afforded by hand-written tests is favourable to the potential for guessing on multiple choice assessments.

In terms of content, Calculus I is designed as a course that reviews many topics from high school calculus while introducing a few new topics and additional rigour; it is a course whose purpose is often framed as "getting everybody on the same page and speaking the same language." Concepts include: a review of functions with an emphasis on trigonometric functions; transformations of these functions; the absolute value function; solving inequalities; solving limits; the formal ε-δ definition of the limit; continuity; the Intermediate and Extreme Value theorems; the definition of the derivative; derivative rules; higher-order derivatives; implicit differentiation including log differentiation; related-rates problems; differential approximation; Fermat’s Theorem, Rolle's Theorem, and the Mean Value Theorem; curve-sketching; optimization problems; basic integration techniques including the method of substitution; Riemann sums; definite integrals and the Fundamental Theorem of Calculus; and finally, applications of integrals including finding areas and solving word problems.

Historically, two to four tests have been set during the semester. Since 2007, these tests have been accompanied by a set of weekly online quizzes, utilizing Maple TA™, that serve as enforced homework. (Maple TA™ is an online learning environment that allows for testing of students in a wide variety of ways.) There is a final exam that has historically carried a weight of between 35 and 50 percent of the overall grade. In recent years, none of these assessments have included any multiple choice questions at all. Rather, they are divided into
two parts: Part A is a "Quick Questions'' section where students answer simple questions or problems by writing their answers in a box at the side of the page. Student work need not be shown, and partial marks for incorrect solutions are limited. Part B is a "Longer Written Answer'' section where full solutions are expected and partial marks are granted if sound mathematical steps are made. Tests are written using brand new questions each year primarily to minimize academic misconduct, which has the added benefit of allowing instructors to use past tests to provide practice resources for students. Students are aware that typically none of the questions on a practice resource will closely resemble problems on the "actual'' test, but this opportunity for extra practice has proved to be a popular idea among students. Dedicated "extension'' problems, intended to be challenging, are included on each assessment. Accounting for 10 to 20 percent of the marks on a test or exam, these questions tend to blend concepts or allow students to explore simple new concepts.

3. What is a Warm-Up Test and how does it compare to a Diagnostic test?

Feedback after the Fall 2013 semester indicated that many students struggled with a few fundamental concepts from the very beginning of the course in September. These concepts (like basic arithmetic, functions, and trigonometry) provide a foundation for many of the topics covered in the course. Thus, it seems reasonable to infer that having weaknesses in the understanding of these fundamentals would make it more difficult to gain understanding of new concepts throughout the rest of the course that depend on this foundation. A student with these foundational weaknesses may be inclined to resort to surface learning rather than deep learning, as has been discussed by Prosser and Trigwell (1999).

The idea of a "Warm-Up Test" was conceived as the result of discussions between the instructors about this very issue. A Warm-Up Test is a test, held early in the semester, that only tests concepts that are assumed to have been learned in prerequisite courses. The tested concepts are chosen to be those topics that will certainly be used to develop the calculus techniques later in the course (functions, trigonometry, etc), and do not include any calculus concepts. Students are aware of the topics that they should be prepared for from the beginning of the semester. The onus is on them to prepare because with the exception of trigonometry, no lectures are dedicated to material covered on the test. (It was felt that, due to the exceptionally important nature of trigonometry in first-year courses, along with high student anxiety surrounding that topic, the instructors would spend lecture time to review this.)

In terms of the logistics and specifics, Warm-Up Tests are held at the end of week 2 of a 12-week semester, outside of class time. They possess the same structure as a "regular test'' (as outlined in section 2); all hand-written with no multiple choice. With respect to content, the Warm-up test contains only prerequisite material (no Calculus concepts), and only tests at a basic level with no extension questions. The Warm-up test carries a small, but significant (to encourage participation) proportion of a student’s final grade (currently 10
percent as compared to 20 percent for regular term tests). This weighting is fully transferrable to the Final Exam should the Final Exam result be higher. This is to take the pressure off of the Warm-up test so that students use it as tool for indicating their readiness, rather than a stressful event. Finally, the Warm-up test is hand-graded and returned to students with written feedback within a very short time (typically one or two days). Detailed solutions are available online to all students immediately following the Warm-Up Test so that students can immediately follow up and learn from their mistakes while the material is fresh in their heads.

There were many motivations for introducing the Warm-Up Test. First, students lose some of their academic learning over a summer holiday. This learning loss has been shown to not only increase as students get older, but hurts mathematics learning more than other school subjects (Kerry, 1998). An early test forces students to hit the ground running and quickly reminds them of some topics that they might be rusty on after a summer vacation.

Next, it has been shown that high-stakes testing may discourage active student learning and may even have negative effects upon the classroom discourse (Wideen et al., 1997). There is incentive for students to do well on the Warm-Up Test, because doing so will make it likely that the more difficult Final Exam is weighted less heavily.

Most obviously, though, by holding this early test, student weaknesses can be identified and very early written feedback can be given by instructors and Teaching Assistants, which may have a significant impact upon student learning (Hattie & Timperley, 2007; Rakoczy, 2013). Since students are told that the Warm-Up Test topics are those that will certainly be used later in the course, students can come to an early realization of the important topics that they are weak in, and seek assistance or put in extra work ahead of the later, more heavily-weighted assessments. The very fast timeframe of grading and returning the Warm-Up Test is enforced to make sure that students have the opportunity to immediately start working on any weaknesses that have been identified.

A Warm-Up Test is fundamentally different from the typical diagnostic tests that have been administered at various universities though the two concepts have similarities. A diagnostic test is typically given ahead of a semester or at the very beginning of a course in order to identify student strengths and weaknesses, but is not typically a test that is given for grades. Often, a diagnostic test does not possess a dedicated focus toward the course to come, instead concentrating on a more abstract and broad set of fundamental mathematical skills. Finally, nearly all diagnostic tests are given in a multiple choice format, precluding any personalized written feedback.
4. An Outline of the Study

As a pilot of this novel assessment method, a Warm-up Test was created by the authors of this manuscript and administered to a first-year Calculus I class. Following the completion of the test, ten teaching assistants, with the help of solutions provided by the authors of this manuscript graded and returned the Warm-up Test. The analysis that follows was conducted after the entire semester was completed. To investigate the validity of this pilot effort, We wish to perform an analysis of grade data in order to help answer a few questions:

- Do students who perform well on the Warm-Up Test tend to perform well throughout the rest of the course?
- Are poor grades on the Warm-Up Test associated with poor grades throughout the course or higher failure rates?
- Does the inclusion of a Warm-Up Test promote student learning and better final exam results than if there was no early assessment given?

To measure this, we performed an analysis of Warm-Up Test grades as compared to Term Test and Final Exam grades, and followed each student in the class through the semester to evaluate their subsequent performance on later assessments. We first compared the Warm-Up Test grades to grades obtained for Test 1; then we compared the Warm-Up Test grades to grades obtained in the Final Exam. Finally, we evaluated performance throughout the course by comparing Warm-Up Test grades to final overall grades. For this final comparison, it is noted that this final overall grade may include and thus be influenced by the grade from the Warm-Up Test itself.

To compare in all cases, assessment results were categorized by letter grade, and we recorded the number of students that moved from each grade category to each other between assessments. We used the following standard groupings:

- 90-100 percent is an A+ grade;
- 80-89 percent is a A grade;
- 70-79 percent is a B grade;
- 60-69 percent is a C grade;
- 50-59 percent is a D grade;
- 49 percent and below is an F grade.

The study included a total of 690 students. Only students who were present for all assessments were used in the analysis (students may have missed an assessment for a variety of reasons, including illness, etc).
5. Methodology

5.1 Comparison: Warm-Up Test Grades and Test 1 Grades
The average grade received on the Warm-Up Test was 79.6 percent while the average grade received on Test 1 was 63.0 percent. We wished to measure the significance of the difference in mean scores. As the data were dependent, paired data was constructed by subtracting each individual student’s Test 1 grade from their Warm-up Test grade. A two-sided paired-t procedure was then administered on this paired data. A p-value of $1.319 \times 10^{-146}$ was obtained, indicating that the student grades for the two assessments were significantly different. Further, a 95 percent confidence interval indicates that student grades were 15.6 - 17.5 percent higher on the Warm-Up Test than on Test 1.

We found that despite the significant difference in grades for these two assessments, the Warm-Up Test results strongly predicted student performance on Test 1. Table 1 provides a detailed description of the data collected. The data describes the number of students to go from receiving any particular grade on the Warmup Test to grades received from future assessments. Percentages given are proportions of those who received the same letter grade on the Warmup Test. For example, from the table, 48 students who received an A on the Warmup Test received a B on Test 1. This represents 23.3 percent of all students who received an A on the Warmup Test.

Of the 690 students that were included in the study, 619 received a lower grade on Test 1 than on the Warm-Up Test. Few students (20 out of 690) failed the Warm-Up Test, receiving an F. Nevertheless, 75 percent of students who received an F on the Warm-Up Test also received an F on Test 1. Similarly, for students who received a D on the Warm-Up Test, 76.7 percent received a grade of D or F on Test 1 while students who received a C on the Warm-Up Test did not fare much better. It is interesting to note that not a single student who received a C or lower on the Warm-Up Test received anything higher than a B on Test 1. Meanwhile, a plurality of students who received a B on the Warm-Up Test received a D on Test 1; of students who received an A on the Warm-Up Test, the greatest number received a C on Test 1; while on Test 1, a B was the most common grade obtained for students who received an A+ on the Warm-Up Test. It is also noteworthy that of the 551 students who received a grade of B or better on the Warm-Up Test, only 8.2 percent went on to receive a grade of F on Test 1.

5.2 Comparison: Warm-Up Test Grades and Final Exam Grades
The average grade received on the Warm-Up Test was 79.6 percent while the average grade received on Final Exam was 68.6 percent. As the data were dependent, paired data was constructed by subtracting each individual student’s Final Exam grade from their Warm-up Test grade. A two-sided paired-t procedure was then administered on this paired data. A p-value of $5.48 \times 10^{-69}$ indicates that again, the results of the two assessments were significantly different. Further, a 95 percent confidence interval indicates that student grades were 9.9-12.1 percent higher on the Warm-Up Test than on the Final Exam.
Table 1. WU = Warmup Test; FE = Final Exam; FG = Overall Final Grade

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<th>Score Transitions</th>
<th>Count (WU to T1)</th>
<th>Proportion (%) (WU to T1)</th>
<th>Count (WU to FE)</th>
<th>Proportion (%) (WU to FE)</th>
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Despite that the Final Exam occurred three full months after the Warm-Up Test,
we nevertheless found that again, Warm-Up Test results provided a strong prediction of Final Examination results.

Of the students who received an F on the Warm-Up Test, 50 percent also received an F on the Final Exam. A slightly greater proportion (58.1 percent) of students who received a D on the Warm-Up Test received an F on the Final Examination. It is a note of interest that a greater proportion of students who received a D on their Warm-Up Test were not successful on the Final Exam than those who received a grade of F on their Warm-Up Test. Students who received a C, B, or A on the Warm-Up Test were widely distributed throughout all of the categories for the Final Exam, with some students failing while others obtained grades as high as an A+, though Final Examination results did generally increase with greater Warm-Up Test results. Students who received an A+ on the Warm-Up Test, however, provided quite a different picture, with over 85 percent of these students receiving a B or higher on the Final Exam; a plurality of such students (35.1 percent) received an A+ on the Final Examination.

5.3 Comparison: Warm-Up Test Grades and Overall Final Grades

The average grade received on the Warm-Up Test was 79.6 percent while the average overall final grade was 71.0 percent. As the data were dependent, paired data was constructed by subtracting each individual student’s Final grade from their Warm-up Test grade. A two-sided paired-t procedure was then administered on this paired data. A p-value of $1.5 \times 10^{-73}$ indicates that once again, the results were significantly different. A 95 percent confidence interval indicates that their Warm-Up Test results were 7.7 - 9.3 percent higher than their final overall grades.

We found that the Warm-Up Test was strongly predictive of overall final grade, to a remarkable extent. Of the 20 people who received an F on the Warm-Up Test, 65 percent of them went on to fail the course with an F. Generally speaking, the better that students did on the Warm-Up Test, the lower the failure rate for the overall course. This effect was by far most visible for those students who received an A+ on the Warm-Up Quiz; only 3 (1.5 percent) out of these 194 students received an overall grade of F in the course, while much higher failure rates were observed for other students. On the other hand, students who received high grades on the Warm-Up Test had much greater chances of receiving an A or A+ in the course. It is interesting, however, that several students who received a D or C on the Warm-Up Test managed to earn an overall grade of A despite their shaky start to the semester.

6. Discussion

Most of the results seem to speak for themselves: The Warm-Up Test is a test of foundational skills, and so a student who struggles with these underlying fundamentals is easily identified with a poor performance on the Warm-Up Test. This weakness strongly predicts a weak performance not only on the first test, but throughout the entire course. We noticed that many students who performed at a C or B level tended to have a mixed range of performance
throughout the course, but generally fared more poorly on future assessments. Students who improved their performance, while relatively few in number, may represent a group of students who recognized weaknesses that were identified on the Warm-Up Test and who subsequently worked to shore up their knowledge. By and large, however, students who did more poorly on future assessments likely did so because their fundamental mathematical skills were still not as solid as for many of their peers. The students who displayed the greatest resilience throughout the course were those who received an A+ on the Warm-Up Test, with a drastically smaller proportion experiencing a large drop in their performance throughout the semester. Perhaps it is the case that those students were "fully ready" to move into university calculus, since it is these students who had already achieved mastery of fundamental concepts. High school grade point average and student attitude have been shown to have positive correlation with performance in a first-year university calculus course (Pyzdrowski et al. 2013) further study in future offerings of the course may reveal whether either or both of these factors could be correlated to Warmup Test scores, as one might expect.

We believe that the Warm-Up Test can be viewed as an important measure that students may use to gauge their readiness to continue forward in Calculus I. Further, based on the results, we feel that instructors may be able to use this past data to help motivate students in future offerings of the course. For the case of those students who receive a B or C on the Warm-Up Test, it is especially evident that a high overall grade is still attainable, though perhaps some remedial work, support and encouragement would be necessary. Students who receive very low Warm-Up grades of D or F might take their result as a warning that they are not ready for some of the rigorous material that will be covered in the course until their foundational skills are improved, and as such urgently require immediate extra support or perhaps will face failure.

The observation was made during this analysis that despite a weak performance on Test 1, the Final Exam (and ultimately, the final grades) represented a marked improvement. The material on the Final Exam was certainly not easier to grasp than the concepts appearing the first test, because the Final Exam was comprehensive and thus included all of the material from Test 1 along with many other topics from the rest of the course. Can this improvement be explained by the presence of the Warmup Test at the beginning of the course, or are there other factors at play? Perhaps this is a question that may be explored in future offerings of the course.

7. Biases

We recognize that there are some potential biases in our results, many of which are unavoidable and inherent to a large first-year university course. These biases could have a significant effect upon comparisons or student performance. In the interest of full disclosure and completeness, we discuss some of these biases here.

- Scheduling of tests and final examination alongside those of other
courses. Students may not be able to prepare as adequately for a test if, by chance, they have another large assessment or due date for a different course very near the date of the assessment for their calculus class.

- Since the Warm-up test weight can be shifted to the final exam, some students may opt not to write the Warm-up Test. These students would therefore not have been included in our study and may have skewed the results that we observed.

- With regards to the comparison between the Warm-Up Test results and the overall final grade, it is the case that for 76.5 percent of students (those who performed better on the Warm-Up Test than on the Final Examination), the overall final grade does incorporate the result from the Warm-Up Test itself, which is weighted at 10 percent of their final grade.

- The results presented in this paper only include results from a single semester, representing the very first time that first-year calculus was presented with a Warm-Up Test as part of the assessment. These results should be considered alongside results from additional future semesters in order to corroborate our findings and present a stronger argument for the value and predictive power of this new kind of assessment.

8. Future Work
Recognizing that the Warm-up Test results were a strong predictor of performance in Calculus I, perhaps greater efforts should be made to help students that are identified as “at risk” by the Warm-up test to bring their skill set up to an appropriate level to encourage their success in the course. A variety of initiatives may be helpful in this regard including:

- **The Development of an Email Feedback Tool**
Entry-level mathematics courses are among some of highest enrollment classes on campus. As a result, students can often feel “like a number” among their peers and classmates. In an effort to personalize student experience and promote individual recognition, we propose the development of an Email Feedback Tool that will allow instructors to generate personalized emails to students regarding their progress or performance. This tool would also allow instructors to provide the student with information about learning resources that they can take advantage of should they be struggling with course material.

- **The Compilation of Feedback Regarding Student Opinions of Preparedness**
With the approval of the Research Ethics Board, we conducted a survey in the Fall 2015 semester in our Calculus I class regarding student opinions. We asked for their opinions of their preparedness for University mathematics, their thoughts on Warm-up test, as well as methods and techniques that they use to study for mathematics tests. We would like to compile and investigate the resulting data from this survey so that it may be used to improve our
understanding of the student perspective of our course (and University as a whole). This information can have a significant impact on the delivery of our course going forward and how we cope with preparedness challenges.

- **The Development of a “Foundational Mathematics” Refresher Course**

The Warm-up test results explored in this paper indicate that while a number of students perform very well on the Warm-up test, many other students perform far below an acceptable level. Beyond simply reviewing concepts on their own, students currently have no designated resource that they can use to catch up. In order to promote the success of our students, we propose the development of a “Foundational Mathematics” Refresher Course to assist students in filling in any gaps in their mathematical backgrounds (perhaps exposed by their Warm-up test). Since entry-level math courses are integral to so many other courses, this would provide a substantial resource for students to increase their skills up to an appropriate level, thus encouraging success in their further studies. Students performing below a designated level would be enrolled in this course on a mandatory basis in order to bring their foundational skills up to the necessary level for success in Calculus I. This course would run concurrently with Calculus I as a three or four-week session, to end in mid-semester. Mandatory testing to ensure mastery of foundational concepts would allow students to remain in their Calculus I course, while failure to pass such testing in this foundational course by its completion would indicate a lack of preparedness and thus would require withdrawal from the course and subsequent remedial work before reinstatement into Calculus I.

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**9. References**


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