

Conquering Worrisome Word Problems – Algebra Success

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Abstract. High school students can struggle with word problems in upper level math classes. Causes for this struggle could include lower reading comprehension, limited mathematic vocabulary, and difficulty changing words to algebraic expressions. This article proposes three techniques to help teachers instruct these struggling students that include (a) organization by difficulty of comprehension and computation (b) scaffolding and (c) utilizing the *Explain, Practice and Assess* (EPA) strategy.

Keywords: math word problems; instruction; high school; struggling students

Introduction

Word problems -- the bane of high school algebra students! Often word problems cause anxiety and confusion, leading to the fear and dislike of mathematics for many high school students (Chapman 2002; Haghverdi & Wiest, 2016; VanSciver, 2008) lasting throughout their mathematics careers. Word problem angst negatively influences how students perceive not only mathematics, but also science, technology, and engineering as well (Didis & Erbas, 2015; Kribbs & Rogsowsky, 2016; Sisco-Taylor, Fung & Swanson, 2014; VanSciver, 2008).

Word problem success is important in terms of algebra because word problems show and model how our physical world can be interpreted and understood using algebra. When students see the practical application of topics used in word problems, they comprehend and become more invested in the mathematics (Chapman 2002; Lim, 2016; Wilburne, Marinak, & Strickland, 2011). This is especially true when dealing with at-risk populations whose understanding of word problems significantly increases when their content is made authentic and culturally relevant (Dominguez, 2016; Wilburne, Marinak & Strickland, 2011).

Mastery of word problems is also linked to success on criterion referenced (standardized) tests (Bates & Wiest, 2004; Fuchs, Compton, Fuchs, Powell,

Schumacher, Hamlet, & Vukovic, 2012; Fuchs, Schumacher, Long, Namkung, Malone, Wang, & Changas, 2016; Hickendorff, 2013; Sisco-Taylor, Fung & Swanson, 2014; Jitendra, Sczesniak, & Deatline-Buchman, 2005; Powell, Fuchs, Cirino, Fuchs, Compton, & Changas, P. C. 2015) and is highly correlated ($r = .37$) with working memory (Peng, Namkung, Barnes, & Sun, 2016), resulting in increased quality of computational skills and algebraic reasoning (Jitendra, Griffin, Haria, Leh, Adams, & Kaduventoor, 2007; Powell & Fuchs, 2014). These abilities are crucial in future mathematics and science classes as these fields require the skills essential to solving word problems.

The word problem hurdle has not been conquered. While there is much literature on elementary (1-6th grade) strategies (Boonen, Van der Shoot, Van Wesel, De Vries & Jolles, 2013) Depaepe, DeCorte, & Verschaffel, 2010; Moreno, Ozoglu, & Reisslein, 2011, Nortvedt, Gustafsson & Lehre, 2016), there is little research on secondary Algebra I (8-12th grade) strategies (Bush & Karp, 2013; Haas, 2005; Jitendra et al., 2013). Since students are still struggling with understanding word problems, it was imperative to find a solution.

One answer to the word problem angst lies in changing our pedagogy - in summary, how word problems are introduced and taught. In secondary education, word problems should be approached as would any other algebraic skill; that is, in an organized *unit*, where word problems are categorized by content (type) and level of difficulty. After a review of current practices and multiyear classroom experience, three problem areas needed to be addressed in the unit: organization, scaffolding, and practice/assessment. Within the unit, word problems should be organized by decoding difficulty (conversions of words to algebraic expressions) and computational difficulty. Another essential component to the solution of word problems is scaffolding. This involves going from the simplest type of word problem to the more difficult in two arenas: variable-identification complexity (predefined to non-defined plus) and relationship complexity (development of the equation). Finally, the *Explain-Practice-Assess* or EPA strategy needs to be utilized. This EPA strategy gives teachers the opportunity to take the class as a whole and make it progress to mastery of word problems; thus, bringing every student along with this learning so every student can succeed.

After a review of current practices, three problem areas were found. These areas are identified below and are followed by a presentation of a viable solution.

Literature Review on Current Word Problem Pedagogy

Many teachers feel ill equipped to handle word problems (Brown, 2012, Chapman, 2002; Depaepe, et al., 2010; Green, 2014; Wright, 2014) and either ignore them or tack a few problems to the end of a lesson (Snarks, 2014). They are often given an abbreviated explanation or algorithm with very little follow-up practice provided (Chapman, 2002; Powell, 2011; VanSciver, 2008). In secondary education, word problems are not approached as would any other algebraic skill - in an organized *unit*, categorized by word problem content (type) and level of difficulty (simple to complex). Instead, word problems are

treated as isolated add-ons to a different topical unit in an effort to show application of the algebraic material taught (Benson, 1994; Burger, 2007; Larson, 1996; McConnell, 1998).

Textbook pedagogy mirrors what has been generally taught in the classroom. In a survey of the major Algebra I textbooks, including *Addison-Wesley*, *McDougal Littell*, *Houfflin Mifflin*, *Hickory Grove*, *Holt*, *Rinehart and Winston*, and *Scott-Foreman*, it was found that textbooks varied widely in the extent to which word problems were explained. The number of exercised examples that were practiced and assessed also varied in the major texts (not including supplemental material). On average, three word problems per content topic were addressed, and these were predominately add-ons at the ends of the lessons.

A major problem with word problems involves reading comprehension, which is largely rooted in vocabulary knowledge. Vocabulary transference was emphasized where words were translated into algebraic expressions (e.g. “and” means add or “of” means multiply). However, students were not internalizing the vocabulary as it was presented (Didis & Erbas, 2015; Haghverdi & Wiest, 2016; Holmes, Spence, Finn & Ingram, 2017). Students were learning a broad range of vocabulary terms, which they, themselves, had to know and appropriately use in a variety of different word problems without sufficient guidance or practice. Because each word problem required its own specific set of words that the students had to identify, success required mastery of a moving target. Students did not have the opportunity to see and appreciate one approach or one set of vocabulary terms before having to apply another. This means that students were not realistically given the chance to achieve mastery.

Consider the following three word problems that demonstrate the difficulties encountered in the current practice of teaching word problems (as explained above):

Word Problem 1: Two more than three times a number is equal to thirty minus that number. Find the number.

Word Problem 2: One complementary angle is ten more than the other. Find the measures of these two angles.

Word Problem 3: Izzie has seven more dimes than nickels. Altogether she has \$2.95. How many nickels and dimes does she have?

In all these word problems there is the vocabulary component – changing words to algebraic expressions and equations. However, word problems should be grouped by considering the degree of transference and computational difficulty. In Word Problem 1, it is more or less simply a translation from words to an algebraic equation. In Word Problem 2, two things must be considered when writing the equation. One consideration is writing the expressions for the two angles involved (x and $x + 10$) and the second is showing how these two angle

expressions are related by using the definition of complementary [$x + (x+10) = 90$]. In Word Problem 3, when money is involved (coins), the expression has to take into account both the number of each coin type [$x = \#$ of nickels; $x+7 = \#$ of dimes] and the value of each coin type [$5x =$ value of nickels; $5(x+7) =$ value of dimes]. The final step requires integrating the value of the coin, the number of the coin, and their sum (total value) into one equation [$5x + 5(x+7) = 295$; cents]. Additionally, students have to take into consideration unit value, recognizing that the equation must either be written in cents or dollars, and the appropriate conversions performed. In word problems that involve money, each quantity that adds up to the sum requires two considerations by the students.

The solution of word problems needs to be treated as a distinct skill. Word problems are a unique blend of practical application, algebraic reading comprehension and computational skills. Traditionally, however, all of these individual skills (comprehension and computation) have usually been lumped together in the handling of word problems. The assumption is made that students can look at these three problems, assess the appropriate approach in each case, and appreciate the essential differences between them. Moreover, this assumption is made of students just beginning their study of algebraic word problems. In order to achieve success in additional word problems, the students would have had to make all of these assumptions correctly - in addition to mastering the computational skills of the lessons. Teachers unintentionally required more of students than they were able to achieve, simply because the complexity of even the simplest set of word problems was not recognized

This resulted in a variety of disjointed word problems at the end of most lessons which supported a lesson's content, but did not aide in students' ability to master solving word problems. Once again, students did not have the opportunity to see and appreciate one approach before having to apply another. Nor were students giving the opportunity to practice and internalize one approach to mastery. By not categorizing word problems by content difficulty, students were presented with a challenge that was impossible for all but the brightest.

A Viable Solution

In summation, after a careful analysis of current teaching practices, three areas in which the approach to word problems can be strengthened were identified:

(a) organization by difficulty of comprehension and computation (including decoding), (b) scaffolding, and (c) the EPA strategy (Explain, Practice, and Assess) (Holmes et al., 2017).

Organization. Organization is the key to a successful approach to introducing and teaching word problems (Holmes et al., 2017). The organization, which groups word problems by type, stresses similarities among the word problems. These similarities are based upon decoding difficulty (conversions of words to algebraic expressions) and computational difficulty as expressed earlier. This is the easiest way for students to internalize the strategies needed to attack a word problem. This approach guides the students in looking at word problems,

selecting appropriate approaches, and appreciating the essential differences among them. Having the student reword the problem using a vocabulary that he or she can fully understand may also help with organization as well as comprehension of the problem. Once an individual group of similar word problems is mastered, the strategy or method used to solve the problem can be applied to new, but similar, word problems. This should enable student success with word problems, reduce anxiety, and greatly diminish negative perceptions of mathematics in general.

As reviewed earlier, most modern algebra texts deal with word problems as a totality, where a smattering of varied word problems appear at the end of an exercise set. Because similarities among these problems are not emphasized, students cannot easily determine/identify the solution method required. Word problems do not appear distinct, separate from one another, and have no common solution pattern (method of solving the problem). By classifying word problems by type, this lack of solution and strategy continuity is eliminated.

Scaffolding. Word problems should start with the simplest type and gradually work up to more difficult problems. Scaffolding is not readily apparent in the traditional treatment of word problems; in most cases, an assortment of word problems of vastly different difficulty levels is attached to the end of a lesson. Within that smattering of word problems, the students are never given the chance to start at the beginning and take simple steps towards the understanding of *how to do* word problems. The students are taught how to approach the content lesson, but not how to approach the solving of word problems in general – the skill that they lack and that needs to be developed.

After extensive study of the word problems often encountered involving one equation and one unknown, one possible organization scheme (Holmes et al., 2017) begins with a variable that is predefined and scaffolds up to a variable that is not predefined and involves additional vocabulary or content knowledge. The following exemplify this progression:

Word Problem 4 (Predefined): Genelle is five less than twice her daughter Rachel's age. If Genelle is 45 years old, how old is her daughter?

Word Problem 5 (Not Predefined): The length of a rectangle is twice the width. The perimeter is 48 inches. Find the length and the width of the rectangle.

Word Problem 6 (Not Predefined-Plus): Aarika is selling raffle tickets: two-dollar tickets for a chance to win an iPad; five-dollar tickets for a chance to win a Dell desktop. Aarika sold twice as many two-dollar tickets as five-dollar tickets. Her total ticket sales amounted to \$45.00. How many two-dollar and five-dollar tickets did she sell?

These three word problems exemplify one possible way to scaffold simple word problems involving one equation with one unknown. These word problems are scaffolded two ways: variable-identification complexity (predefined to non-predefined plus) and relationship complexity (development of the equation).

Variable-identification Complexity. Variable identification complexity (predefined to non-predefined plus) involves expressing one variable in terms of another and identifying the relationship between the two expressions. However, at the simplest, predefined level, the relationships are given (defined); all necessary information is stated in the problem. The relationship between the two variable expressions can be found within the problem.

Example #4 involves the expressions x and $2x-5$. These expressions were based only on information given in the problem; no other relationships needed to be used (predefined). These are the simplest word problems in this category.

At the more complex non-predefined level, the equation is based upon additional information, most often a matter of algebraic vocabulary such as complementary/supplementary or geometric vocabulary such as perimeter. It can also involve the complex relationship between items of different monetary value. In order to solve these problems, students must make use of information not explicitly stated in the problem.

Examples #5 and #6 are both non-predefined word problem types. In Example #5 the additional information required is the definition of perimeter, and it must be used to set up the equation: $w=x$, $l=2x$, $p=2L + 2w$; $48=2(x) + 2(2x)$.

Example #6 requires an understanding of the relationship between items of different monetary value. x = number of \$5 raffle tickets, $2x$ = number of \$2 raffle tickets. So, students must understand how the monetary value of the tickets sold determines the final equation: $5x + 2(2x) = \$45$

In terms of scaffolding (difficulty level), problems involving money are more complex than problems requiring additional vocabulary.

In this sequence of word problems, students moved from the simplest to a more difficult form.

Relationship Complexity. Relationship complexity or development of the equation refers to the degree of complexity involved in the relationship between the two expressions for the quantities identified in the problem. In Example #4, the simplest word problem, the two quantities are given by the expressions: x =Rachel's age, $2x-5$ = Genelle's age. The wording of the problem indicates that Genelle's age is 45. Translating that, the equation becomes $2x-5 = 45$. Example #5 is a slightly more complex word problem in that the definition of perimeter ($2w + 2l = p$) is required. Substituting x for the width and $2x$ for the length, the

final equation becomes $2x + 2(2x) = 48$. The final example is the most complex, requiring the monetary relationship between the total value of the two-dollar tickets and the five-dollar tickets: $5x$ equals the monetary value of the \$5 tickets and $2(2x)$ equals the monetary value of the \$2 tickets; their sum is \$45, resulting in the equation $5x + 2(2x) = \$45$. In addition, care must be given to keep all monetary values in either dollars or cents, especially when introducing this level of complexity.

Notice, that in this sequence of word problems computational vocabulary is kept simple. Twice ($2x$) was used in each level of word problem difficulty and calculations are kept simple. Hence, the increased difficulty results from the increased complexity involved in the relationship of the expressions for the quantities used in each problem. The challenge of word problems encountered by the students is not exacerbated by computational difficulties.

Explain-Practice-Assess (EPA) Strategy. As each individual category of word problems is introduced, the approach should be explained in detail as the example problems are being solved. As evidenced-based practice dictates, a good explanation involves three steps: (a) the teacher explains one or two examples in detail as s/he models the solution; (b) the third and fourth examples are completed with teacher-prompted student involvement (guided instruction); (c) the fifth and sixth examples are student-led. The number of examples in each step is situationally determined. An advanced class may only need two examples; while an at-risk class may require more. In addition, student questions should be strongly encouraged at each level. At level c, the teacher should monitor each student with the goal of having the entire class reach a basic level of understanding (to the extent possible). This is done prior to allowing students to individually practice the material. This *Explain-Practice-Assess (EPA)* strategy gives teachers the opportunity to take the class as a whole and make it progress through the material, leaving no child behind.

Multiple practice exercises should be provided, so that the students can practice what they are learning discretely, meaning the students are given the opportunity to master each level of word problem before proceeding to the next level. Three practices are suggested. With the first practice, students will make a variety of mistakes; this is to be expected. In the second practice, students have corrected the previous errors and perhaps make new ones. In the third practice, the hope is that students will have mastered this limited lesson – the one type of word problem introduced. Should a fourth practice be required, the first practice can be re-used. In this way, students are very clearly given the opportunity to master the material at each step, leading to success and a positive attitude toward word problems.

The final step involves assessment to determine level of mastery. The assessment should mirror the practices. The only real hurdle in the EPA strategy is to harness the involvement of the student. As long as the students are engaged in the process, mastery is assured. If students practice one thing, repeatedly, with teacher monitoring, they will succeed.

By classifying word problems based upon similar strategies and teaching each type in succession, students begin to recognize patterns which facilitate comprehension of the words; they see how each type of word problem can be written algebraically. When word problems are not categorized, but en masse, with every word problem being different, students have a harder time recognizing and then attacking the problems. The repetition and categorizing of the word problems assist the learning process.

As always the numbers used in these word problems are kept manageable. This facilitates understanding rather than time spent on challenging arithmetic. Unfortunately this may lead some students to guess at the answer, bypassing the equation altogether. For each type of word problem, the variable must be identified; the equation must be stated; and the question must be answered. Insistence on these three steps prevents students from taking a shortcut that will harm them when presented with more complex word problems later.

The following is a graphic organizer that summarizes this treatment of word problems involving one equation and one unknown. Word problems dealt with in this manner will have been broken down, categorized, scaffolded, explained and practiced so that student success is assured. Students will complete these graphic organizers where the last two columns will need to be filled out by the students (see Table 1). Please note that in each case, it is essential for each student to write the equation even if it is possible to guess the correct answer.

Table 1: Word Problem Classification Graphic Organizer

Relationship values pre-defined	Example	Variable Identification	Pattern & Attack	Answer
Number Equality	If five less than 6 times a number is equal to 10 more than 3 times a number, what is the number?	$x =$ the number	$6x - 5 = 10 + 3x$	$x = 5$
Consecutive Numbers				
consecutive	The sum of 3 consecutive numbers is 54. What are the numbers?	$x =$ the first consecutive number	$x + x+1 + x+2 = 54$	$x = 17$ 17, 18, 19
even consecutive	The sum of three <u>even</u> numbers is 78. What are the numbers?	$x =$ the first consecutive even number	$x + x+2 + x+4 = 78$	$x = 24$ 24, 26, 28
odd consecutive	The sum of three <u>odd</u>	$x =$ the first consecutive odd	$x + x+2 + x+4 = 99$	$x = 31$ 31, 33, 35

	numbers is 99. What are the numbers?	number		
<i>Note: Odds and evens work the same; this will have to be explained.</i>				
Relational Values not predefined	Example	Variable Identification, Pattern & Attack	Answer	
Sums of numbers	One number is 12 more than another. Their sum is 32. What are the numbers?	$x = \text{one number}$ $x + 12 = \text{another number}$ $x + x + 12 = 32$	10, 22	
<i>Note: In word problems of this type, the first sentence often defines quantities, while the second sentence defines the relationship of the quantities. It is critical in setting up these word problems that the explanation includes defining the second quantity (e.g. $x + 12$) in terms of the first (e.g. x).</i>				
Area and Perimeter	The length of a rectangle is seven more than the width. The perimeter is 54. Find the length and width of the rectangle.	$x = \text{the width}$ $x + 7 = \text{the length}$ $2x + 2(x+7) = 54$	width = 10 length = 17	
Angles				
complementary	Two angles are complementary. One angle is twenty more than the other. Find the measures of these two angles.	$x = \text{one angle}$ $x + 20 = \text{its complement}$ $x + x + 20 = 90$	55°, 35°	
supplementary	Two angles are supplementary. One angle is twice the other. Find the measures of these two angles.	$x = \text{one angle}$ $2x = \text{its supplement}$ $x + 2x = 180$	60°, 120°	
Relational Values not predefined plus	Example	Variable Identification, Pattern & Attack	Answer	
Money Problems involving quantities which have different monetary values	Peppermint patties cost 25 cents each. Jaw breakers cost 35 cents each. Starving Adele wants to buy 15 pieces of candy for \$4.55. How many peppermint patties and jawbreakers can she purchase?	$x = \text{the number of peppermint patties}$ $15+x = \text{the number of jaw breakers}$ $.25x + .35(15+x) = \$4.55$	8 jawbreakers 7 peppermint patties	
<i>Note: It may be easier for students to work in cents whenever possible, thus avoiding decimals. This last equation would then become $25x + 35(15 - x) = 455$.</i>				

Conclusion

In support of this treatment of word problems, there is anecdotal evidence available. One of the authors has used this pedagogy for over eight years and has met with substantial success. Students have achieved significantly better mastery of word problems and no longer avoid them. Students no longer struggled or expressed frustration and dislike for the word problems. Classroom assistants, including one who worked with Algebra I students for many years, commented that this method of tackling word problems gave students an opportunity to experience success not otherwise found. This assistant saw that these students were understanding word problems, doing well on assessments, and displaying a much more positive attitude than in the past. This method has been explained in detail in the textbook, *Now, I Can Understand Algebra*, and is being piloted in several schools in western Michigan. While the success has been mostly anecdotal, this next step will provide empirical evidence.

It is important to note that the strength of this method of teaching word problems is not in that the students memorize types of word problems, but that the students are given scaffolded word problems of differing types in order to be able to better classify and learn how to attack the word problems. The key in this particular method is teaching the students how to breakdown and analyze word problems -- a requisite skill needed in mathematics generally. While empirically, students grades have risen using this method, the true key to success is that students were understanding the process and using the process to attack other word problems such as two equation, two unknown types.

In this article, examples were provided for one equation, one unknown word problem types, but this same treatment (categorizing, scaffolding, EPA) can be applied to many different kinds of word problems (e.g., functions including linear, quadratic, and cubic; two equations, two unknowns; and percentages). The word problem unit described here gives students the opportunity to develop word problem skills from the beginning and provides a good foundation for future word problem study. These skills can be transferred to more complex problems, which involve applying strategies to new concrete and abstract situations.

References

- Bates, E. T., & Wiest, L. R. (2004). Impact of Personalization of Mathematical Word Problems on Student Performance. *The Mathematics Educator*, 14(2), 17-26.
- Benson, J. (1994). *Gateways to algebra and geometry: an integrated approach*. Evanston, IL: McDougal, Littell.
- Boonen, A. J. H., Van der Schoot, M., Van Wesel, F., De Vries, M. H., & Jolles, J. (2013). What underlies successful word problem solving? A path analysis in sixth grade students. *Contemp. Educ. Psychol.* 38, 271-279. doi: 10.1016/j.cedpsych.2013.05.001
- Brown, C. E. (2012). *Answering the call to teach: a novel approach toward exceptional classroom instruction*. Lanham, MD: Rowman & Littlefield Education.
- Burger, E. B. (2007). *Holt algebra 1*. Orlando, FL: Holt, Rinehart and Winston.
- Bush, S. B., & Karp, K. S. (2013). Prerequisite algebra skills and associated misconceptions of middle grade students: A review. *Journal of Mathematical Behavior*, 32(3), 613-632.
- Chapman, O. (2002). Teaching Word Problems: What High School Mathematics Teachers Value. *PME Problem Solving*, 1349-1360.
- Depaepe, F., De Corte, E., & Verschaffel, L. (2010). Teachers' metacognitive and heuristic approaches to word problem solving: analysis and impact on students' beliefs and performance. *ZDM Math. Educ.* 42, 205-218. doi: 10.1007/s11858-009-0221-5
- Didis, M. G., & Erbas, A. K. (2015). Performance and Difficulties of Students in Formulating and Solving Quadratic Equations with One Unknown. *Educational Sciences: Theory & Practice*, 15(4). doi:10.12738/estp.2015.4.2743
- Dominguez, H. (2016). Mirrors and Windows into Student Noticing. *Teaching Children Mathematics*, 22(6), 358. doi:10.5951/teachmath.22.6.0358
- Foerster, P. A. (1999). *Algebra I: expressions, equations, and applications: teacher's edition*. Menlo Park, CA.: Addison-Wesley Pub. Co.
- Fuchs, L. S., Compton, D. L., Fuchs, D., Powell, S. R., Schumacher, R. F. Hamlet, C. L., & Vukovic, R. K. (2012). Contributions of domain-general cognitive resources and different forms of arithmetic development to pre-algebraic knowledge. *Developmental Psychology*, 48(5), 1315-1326. doi: 10.1037/a0027475
- Fuchs, L. S., Schumacher, R. F., Long, J., Namkung, J., Malone, A. S., Wang, A. & Changas, P. (2016). Effects of Intervention to Improve At-Risk Fourth Graders' Understanding, Calculations, and Word Problems with Fractions. *The Elementary School Journal*, 116(4), 625-651. doi:10.1086/686303
- Green, E. (2014, July 23). Why Do Americans Stink at Math. *The New York Times magazine*. Retrieved from https://www.nytimes.com/2014/07/27/magazine/why-do-americans-stink-at-math.html?_r=0
- Haghverdi, M., & Weist, L. R. (2016). The effect of contextual and conceptual rewording on mathematical problem-solving performance. *The Mathematics Educator*, 25(1), 56-73.
- Haas, M. (2005). Teaching Methods for Secondary Algebra: A Meta-Analysis of Findings. *NASSP Bulletin*, 89(642), 24-46. doi:10.1177/019263650508964204
- Hickendorff, M. (2013). The effects of presenting multidigit mathematics problems in a realistic context on sixth graders' problem solving. *Cogn. Instr.* 31, 314-344. doi: 10.1080/07370008.2013.799167
- Holmes, V.L., Spence, K., Finn, J., & Ingram, S. (2017). *Now I Can Understand Algebra!* (Vol. 1 & 2, Algebra 1). Ronkonkoma, NY: North American Linus Publications.
- Jitendra, A. K., Griffin, C. C., Haria, P., Leh, J., Adams, A., & Kaduvettoor, A. (2007). A comparison of single and multiple strategy instruction on third-grade students'

- mathematical problem solving. *Journal of Educational Psychology*, 99(1), 115-127. doi: 10.1037/0022-0663.99.1.115
- Jitendra, A. K., Sczesnink E., & Deatline-Buchman, A. (2005). An exploratory validation of curriculum-based mathematical word problem solving tasks as indicators of mathematics proficiency for third graders. *School Psychology Review*, 34, 358-371
- Kribbs, E., & Rogowsky, B. A. (2015). A Review of the Effects of Visual-Spatial Representations and Heuristics on Word Problem Solving in Middle School Mathematics. *International Journal of Research in Education and Science*, 2(1), 65. doi:10.21890/ijres.59172
- Larson, R. (1996). *Heath passport to algebra and geometry: an integrated approach*. Evanston, IL: McDougal Littell.
- Lim, K. H. (2016). Fostering Algebraic Understanding through Math Magic. *The Mathematics Teacher*, 110(2), 110. doi:10.5951/mathteacher.110.2.0110
- McConnell, J. W. (1998). *Algebra*. Glenview, IL: Scott Foresman/Addison Wesley.
- Moreno, R., Ozogul, G., and Reisslein, M. (2011). Teaching with concrete and abstract visual representations: effects on students' problem solving, problem representations, and learning perceptions. *J. Educ. Psychol.* 103, 32-47. doi: 10.1037/a0021995
- Nortvedt, G. A., Gustafsson, J., & Lehre, A. W. (2016). The Importance of Instructional Quality for the Relation Between Achievement in Reading and Mathematics. *IEA Research for Education Teacher Quality, Instructional Quality and Student Outcomes*, 97-113. doi:10.1007/978-3-319-41252-8_5
- Peng, P., Namkung, J., Barnes, M., & Sun, C. (2016). A meta-analysis of mathematics and working memory: moderating effects of working memory domain, type of mathematics skill, and sample characteristics. *Journal of Educational Psychology*, 108(4), 455-473.
- Powell, S. R. (2011). Solving Word Problems Using Schemas: A Review of the Literature. *Learning Disabilities Research & Practice*, 26(2), 94-108. doi:10.1111/j.1540-5826.2011.00329.x
- Powell, S. R., & Fuchs, L. S. (2014). Does Early Algebraic Reasoning Differ as a Function of Students' Difficulty with Calculations versus Word Problems? *Learning Disabilities Research & Practice*, 29(3), 106-116. doi:10.1111/ldrp.12037
- Powell, S. R., Fuchs, L. S., Cirino, P. T., Fuchs, D., Compton, D. L., & Changas, P. C. (2015). Effects of a multitier support system on calculation, word problem, and prealgebraic performance among at-risk learners. *Exceptional children*, 81(4), 443-470.
- Sisco-Taylor, D., Fung, W., & Swanson, H. L. (2014). Do Curriculum-Based Measures Predict Performance on Word-Problem-Solving Measures? *Assessment for Effective Intervention*, 40(3), 131-142. doi:10.1177/1534508414556504
- Snarks, S.D. (2014). Word Problems Should Be Given at The Start of Lesson, Studies Say. *Education Week*. 34(13), 10-11.
- VanSciver, J. H. (2008). Working on Word Problems. *Principal Leadership*, 8, 6-7.
- Wilburne, J. M., Marinak, B. A., & Strickland, M. J. (2011). Addressing Cultural Bias. *Mathematics Teaching in the Middle School*, 16(8), 461-465.
- Wright, J. E., (2014) An Investigation of Factors Affecting Student Performance in Algebraic Word Problem Solutions. Gardner-Webb University Digital Commons. Education Theses, Dissertations and Projects. Retrieved from http://digitalcommons.gardner-webb.edu/cgi/viewcontent.cgi?article=1025&context=education_etd