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A Socio-Constructivist Perspective on Problem-Solving Approaches in Mathematics: Perceptions of Future Primary Education Teachers

Daniela Olivares*
University of La Serena, Chile

Abstract. This study investigates the perceptions of future primary education teachers regarding the implementation of a problem-solving teaching approach from a socio-constructivist perspective. Although previous research has shown the benefits of problem-solving in mathematics education, few studies have yet focused on preservice teachers in primary education, particularly within a socio-constructivist framework. Therefore, this study addresses this gap by exploring the effectiveness and challenges of the Building Thinking Classrooms approach developed by Peter Liljedahl. The study was conducted through a case study design involving 36 first-year preservice teachers enrolled in a mathematics didactics course at a public university in Chile. Qualitative and quantitative data were collected through surveys, field logs, and classroom observations. The data analysis included descriptive statistics and content analysis to understand the students' perceptions of problem difficulty, engagement, and the overall approach. The findings reveal that the problem-solving approach was positively perceived by the participants, highlighting its role in fostering critical thinking, collaboration, and a deep understanding of mathematical concepts. However, challenges were identified, particularly regarding group work and the autonomy required in solving problems. Furthermore, the study identified areas in which additional support could enhance the effectiveness of problem-solving approaches. This research underlines the importance of integrating problem-solving approaches into teacher education programmes and highlights the need for future research on the long-term impacts and strategies needed to support autonomy and collaborative learning.

Keywords: initial teacher training; motivation theories; problem-solving; socio-constructivist approach

* Corresponding author: *Daniela Olivares, dolivares@userena.cl*

1. Introduction

The sustainability of any educational approach depends on the levels of support and acceptance it receives from the teachers who will implement it (Delgado-Rodríguez et al., 2023; Martinez-Roig et al., 2023). Therefore, in order to encourage the long-term adoption of a new approach and integration into the curriculum, it is essential to understand and address the perceptions of future teachers (Darling-Hammond, 2010). Indeed, preservice teachers' perceptions are crucial for evaluating the relevance and effectiveness of educational approaches as they will, in turn, inform the continuous development and refinement of these approaches (Willis et al., 2021). If future teachers believe in the benefits of an approach, they are more likely to implement it enthusiastically and effectively in their classrooms (Moyer-Packenham & Westenskow, 2013). Conversely, negative perceptions can lead to partial or inconsistent implementation, reducing the overall effectiveness of the approach (Cohen & Hill, 2001).

Regarding the type of approach that future teachers should experience in the area of mathematics, various studies emphasise the importance of those based on problem-solving. Such approaches allow future teachers to better understand the dynamics of constructing mathematical knowledge and making connections between concepts (Barana et al., 2019). Research shows that problem-solving approaches improve conceptual understanding as well as performance in mathematics (Lester & Cai, 2016). Additionally, these approaches are essential for developing critical thinking skills (Darmawati & Mustadi, 2023; Nurhayati et al., 2024) and align with current educational reform expectations, such as justifying mathematical reasoning and emphasising conceptual understanding (Bailey & Taylor, 2015).

Despite the widespread recognition of the importance of these approaches, gaps remain in the research regarding the perceptions of future primary education teachers. Previous studies, such as those by Jiang et al. (2022), have explored the perceptions of future mathematics teachers on problem-solving, but none of these studies to date have focused on socio-constructivist approaches or immersive problem-based teaching experiences. On the other hand, studies such as that by Barana et al. (2019) have included teachers from different specialties with more than 10 years of experience, leaving a gap in the research on preservice teachers.

Therefore, this study explores the perceptions of future primary education teachers regarding the implementation of a problem-solving-based teaching approach from a socio-constructivist perspective; specifically, the Building Thinking Classrooms approach by Liljedahl (2020). Through a case study employing a mixed methods approach (Hernández-Sampieri et al., 2014), it analyses students' evaluations of this approach, focusing on its ability to foster critical thinking, collaboration, and a deep understanding of mathematical concepts. The case study corresponds to a course on Didactics for Teaching Numbers and Operations, in the first year of a degree course in primary education pedagogy, in Chile. During this course, six sessions of the Building Thinking Classrooms approach were implemented, each at the beginning of a learning unit. Both qualitative and quantitative data were used in the case study, with a focus on gaining an in-depth understanding of pedagogy students' perceptions of this approach from a socio-constructivist perspective.

The contribution of this work lies in the fact that, although there is research on teachers' perceptions of problem-solving approaches, most of it has thus far focused on experienced teachers or future mathematics specialists. Therefore, this research aims to help bridge a significant gap by focusing on future primary education teachers, a group that has not been sufficiently studied in this context. Furthermore, while the constructivist approach has been investigated, the implementation of specific frameworks – such as that studied in this work, which prioritises critical thinking and collaborative work in the classroom – represents an innovation compared to previous, more general studies. Lastly, this research also addresses the specific challenges that future teachers face when adopting problem-solving approaches, such as group work and the autonomy it requires, which have not been explored in depth in the existing literature. Through this exploratory study, practical recommendations are sought to improve teacher training programmes and expand the problem-solving approach to include the training of primary education teachers.

2. Theoretical Framework

2.1 Socio-Constructivist Teaching of Mathematics

From the perspective of mathematics, there are three principles that underpin constructivist teaching and learning, which are explained as follows: 1) Students are active participants in learning. Educators should promote tasks that engage students in authentic and complex problems, allowing them to reason and devise strategies; 2) Learning depends on students' prior knowledge. Constructivist instruction aims to challenge students' mathematical reasoning and understanding during problem-solving activities to motivate them to alter their pre-existing schemas and adapt them to new information; 3) Knowledge construction occurs through students' social interactions with more knowledgeable others (Lee et al., 2022). In this regard, the notion of the Zone of Proximal Development (Vygotsky, 1978) is used to highlight the importance of collaborative work in discussions and demonstrations with peers.

In mathematics education research, other theories have also followed socio-constructivist principles and have served as complements to explain specific aspects of mathematical teaching and learning. For example, the Flow Theory (Csikszentmihalyi, 2003) describes the mental state of *flow* as an experience of complete absorption in an activity, characterised by total concentration, a sense of control, and intrinsic motivation, among other aspects. In other words, the activity is so enjoyable and rewarding in itself that there is no need for external rewards (Nakamura & Csikszentmihalyi, 2002). Other theories, such as the Self-Determination Theory (Ryan & Deci, 2000) and the Social Cognitive Theory (Bandura, 1987), also highlight the affective aspect and the importance of motivation in human activity and have been applied in various educational research studies.

The integration of these constructivist and motivational theories provides a robust framework for understanding how students interact with mathematical problems and develop their skills. In this context, problem-solving not only becomes an essential pedagogical tool but also a means by which to promote deep and meaningful learning. By exploring the nature of problems and the processes involved in their resolution, we can identify effective strategies to foster critical thinking and collaboration in the classroom.

2.2 Problems and Problem-Solving

In mathematics education, problem-based teaching is intrinsically related to social-constructivist teaching (Frederick-Jonah, 2022). The concept of a problem has been attributed various definitions throughout history. According to Duncker (1945), a problem arises when a living being has a goal but does not know how to achieve it, thus requiring thought and devising some mediating action between the current state and the desired state. However, Schoenfeld (1985) contends that a problem is a particular relationship between a subject and a task, which makes that task represent a difficulty (intellectual, rather than computational) for that subject. Mason (2015, 2016) draws from Schoenfeld's ideas, considering the problematic nature of problems. Mason suggests that problems do not pre-exist and merely arise. Referring more to the psychological state of a person who experiences a task as problematic, Mason (2015) also incorporates notions from Vygotsky and other psychologists to establish the difference between task, activity, and problem. In his work, he often includes examples of sequences of mathematical tasks, asking: At what point does a subject engage in one of the tasks (stimulus), turning it into an activity (cognitive)? At what point does a subject come to perceive an activity as problematic (as a problem)? Upon reaching this state, all aspects of the psyche would be involved: cognition, affect, behaviour, and attention (Mason, 2015). Furthermore, Liljedahl et al. (2016) add the concept of insight, an element that can help the solver to find solutions creatively.

In reviewing the research in problem-solving up to the 1980s, Schoenfeld (1992) established a framework that takes into account several key aspects: foundational knowledge; problem-solving strategies; monitoring and control; beliefs and affects; and practices that promote mathematical thinking. This framework has served as the basis for numerous current theoretical developments, especially in school-level problem-solving research (Santos-Trigo, 2024). In more recent work, Schoenfeld (2020) has addressed problem-solving as a key component of mathematical thinking, emphasising the centrality of students' thinking in classroom discourse. Once again, the author highlighted the need for students to engage with mathematics as a mathematician would—through inquiry and sense-making (Schoenfeld, 2023).

On the basis of Schoenfeld's model, other authors have proposed extensions and empirical analyses. For instance, Rott et al. (2021) used Schoenfeld's model and empirical data to propose a descriptive model of problem-solving phases. Tay and Toh (2023) developed a problem-solving model focused on the planning stage, expanding support in the intermediate stages to help teachers teach problem-solving with the goal of enabling students to solve problems independently outside the classroom. Recent research has also explored specific dimensions of problem-solving, such as its creative aspects (Jäder, 2022) and the difficulties school students face when solving problems (Säfström et al., 2024).

2.3 Didactic Approaches for Teaching through Problem-Solving

In light of this research, various approaches have been developed to support teaching through problem-solving. One notable example is problem-based learning (PBL), a constructivist instructional approach that challenges students to seek solutions to real-world problems, fostering self-directed learning and strengthening critical thinking skills (Pepper, 2009). In PBL, students work in small groups to construct knowledge based on real-life problems, activating prior knowledge and generating ideas (Tan, 2021). This learner-centred approach empowers students to integrate theory and practice, conduct research, and apply their knowledge to develop viable solutions (Savery, 2006).

Another well-structured approach with its own framework is the Building Thinking Classrooms framework, developed by Peter Liljedahl. This framework provides an approach to creating classroom environments that foster problem-solving and critical thinking (Liljedahl, 2018, 2020). The Building Thinking Classrooms framework is based on the following principles for classroom practice, with the first three being essential components: 1) Provide tasks that make students think. 2) Form random groups. 3) Use vertical non-permanent surfaces (whiteboards). 4) De-front the classroom (stop centring activity at the front of the class). 5) Only answer questions that aim to further thinking (do not give the answer). 6) Deliver tasks verbally and standing. 7) Give students opportunities to check their understanding. 8) Mobilise knowledge (allow groups to observe each other's work). 9) Build and maintain flow through hints and extensions. 10) Consolidate from the base. 11) Encourage students to take meaningful notes. 12) Formatively assess valuable aspects. 13) Help students to know where they are and where they are going. 14) Grade according to data (Liljedahl, 2020).

As can be seen, this approach aligns with the characteristics of constructivist learning, as it promotes students' participation in their own learning process, encourages collaboration and dialogue among peers, and is based on authentic problem-solving that challenges prior knowledge and stimulates the development of new understandings. By placing the student at the centre of the educational process and recognising the importance of social interactions and intrinsic motivation, a dynamic and meaningful learning environment is created. Not only does this approach facilitate the acquisition of mathematical knowledge but it also prepares students to face and solve complex problems in various real-life contexts.

3. Research Objectives

3.1 General Objective

To explore the perceptions of future primary education teachers regarding the implementation of a problem-solving-based teaching approach from a socio-constructivist perspective, by analysing three key dimensions: context description; perception of solved problems; and perception of the approach.

3.2 Specific Objectives

1. To analyse the characteristics of the problems used during the implementation of the problem-solving approach, focusing on their origin, mathematical procedures, number of possible solutions, and the types of representations they allow.
2. To examine the perceptions of preservice teachers regarding the difficulty level, interest generated, contribution to learning, enjoyment, and promotion of collaborative work in the problems solved during the Thinking Classroom sessions.
3. To assess the overall perception of the problem-solving approach from a socio-constructivist perspective, considering its contribution to learning, the difficulties encountered, and its effectiveness compared to traditional teaching methods, as well as its perceived impact on teacher training.

4. Method

4.1 Case Description

The course in which the approach was implemented was Didactics for the Teaching of Numbers and Operations. It is offered in the first term of the Pedagogy in General Basic Education degree course at a public university in Chile. Its purpose is to generate methodological teaching strategies for the numbers and operations contents of mathematics teaching in primary education. Specifically, the course addresses topics related to learning the fundamentals of the decimal numbering system, addition, subtraction, multiplication, and division, fractions, rational numbers, ratios, proportions, and percentages. The entire course spans 16 weeks of classes and two weeks of exams. The implementation of the approach took place at the beginning of the learning units within the course. Although the students had not received prior instruction in problem-solving during this course, it is likely that they had received instruction on some of the topics during their school years.

4.2 Description of the Participants

This study involved 36 future teachers (including 7 men) from the Didactics for the Teaching of Numbers and Operations course, which is part of a Primary Education Pedagogy degree in Chile. The course is taken in the first year of the degree programme and is characterised by its diverse student body; two students in the course are autistic, six have been diagnosed with attention deficit disorder, and one with dyslexia. Of the 36 students, 16 had completed their secondary education in public schools, 18 in private schools with public funding, and two in private schools with private funding. Half of the students are from the province in which the university is located, and the rest are from other provinces.

Additionally, several students have begun studies in other degree programmes, both within the university itself and at other institutions.

4.3 Study Design

The research follows a case study design. A case study is a type of design that focuses on the description and in-depth examination or analysis of a unit and its context in a systematic and holistic manner (Hernández-Sampieri et al., 2014). In education, case studies involve in-depth investigations of specific phenomena, such as individuals, classes, or institutions, using observations, interviews, and document analyses (Traser, 2016). Providing rich, descriptive interpretations, they enable researchers to hear the voices of participants (Mendaglio, 2003). The research process involves defining the problem, selecting a case, handling theory and literature, and mastering data collection methods (Merriam, 1988). This type of design was selected because its purpose is to examine a particular situation in depth using a variety of data collection methods (Merriam, 1988). According to its purpose, the study corresponds to an intrinsic case study, as its aim is not to build a theory, seek any kind of generalisation, or represent other cases, but to explore and understand the uniqueness of the case (Grandy, 2009). The case corresponds to a course of 36 students on Didactics for Teaching Numbers and Operations, in the first year of a degree course in Primary Education Pedagogy, in Chile.

Case studies focus on one unit of study and can explore complex environments with varying dimensions (Erickson, 2020). According to this, the study considered three dimensions, each comprising of various variables that allowed for an in-depth exploration of the case. Table 1 presents the dimensions, variables, types of data, and analyses performed.

Table 1: Summary of the dimensions, data, and analyses performed

Dimension	Variable	Types of data collected	Item type	Type of analysis
1. Context description	Characteristics of the problems Classroom observations	Qualitative	Analysis matrix Field log	Content analysis Content analysis
2. Perception of the solved problems	Difficulty Interest generated Contribution to learning Enjoyment Collaborative work	Quantitative	Ranking item	Descriptive statistics
3. Perception on approach	Contribution to learning More difficult aspects Preferred aspects Contribution compared to a traditional class Contribution to teacher training	Qualitative	Open-ended question	Content analysis (emerging categories)

Regarding the Context Description dimension, the problems studied in the six Thinking Classroom sessions were analysed using an analysis matrix. This matrix considered the problem itself, the type of problem according to its origin (curricular or recreational), a description of the mathematical procedure it involves, the type of problem according to the number of possible solutions, and the types of representations that can be used to solve it. To complement the description of the context in which the sessions took place, a field log was kept, which included a record of quotes made by the students while solving the problems. Content analysis was used to analyse the quotes, identifying categories related to the students' perceptions of the problems and the approach in general. Since this variable related to the context was included to deepen the understanding of variables 2 and 3, its results and illustrative quotes are included in their respective sections.

In terms of the Perceptions of the Problems Solved dimension, the variables were considered based on the theoretical framework, which is grounded in the concept of socio-constructivist teaching. To analyse this dimension, a survey including a ranking item was applied. In this type of item, participants must order a list of elements according to a specific characteristic, such as preference, difficulty, importance, etc. Each position in the order represents an ordinal value, with a higher score indicating a stronger preference or a more positive evaluation of the measured characteristic. Based on its ranking, each problem was assigned a score from 1 to 6. This type of question was selected because it is useful for understanding participants' relative preferences and allows for a comparison of the evaluations of different elements within the same set (Krosnick & Presser, 2010). The responses provided an overview of the problems and the preferences of each participant.

With regard to the Perceptions on Approach dimension, open-ended questions were included in the same survey, which were later analysed using content analysis. The objective of this technique is to identify the presence of certain concepts within the content for analysis (Krippendorff, 2019). For this, two assistants were enlisted to analyse the responses independently. Upon comparing the observations of both assistants, an initial agreement of 88% was obtained. Cases with no agreement were discussed together with the author until 100% agreement was reached.

5. Results

The following sections present the results obtained from the analyses of each dimension.

5.1 Contextual Knowledge: Characteristics of the Problems

The following sections present the problems used in the Thinking Classroom sessions, along with a description of the characteristics found in the problem analysis matrix.

- *P1. Boiling eggs*: I have a 4-minute hourglass and a 7-minute hourglass. How can I use these hourglasses to boil an egg for exactly 9 minutes?

This problem is derived from recreational mathematics. It is non-curricular and was used in the first session in which the Thinking Classroom approach was introduced. With multiple possible solutions, there are also multiple systems of representation that can be used to find the solution, using logical reasoning. For example, a number line can be used to determine at what minute and in what way to cook the eggs to meet the condition specified in the problem. One example is the solution shown in Figure 1, but other solutions are also possible.

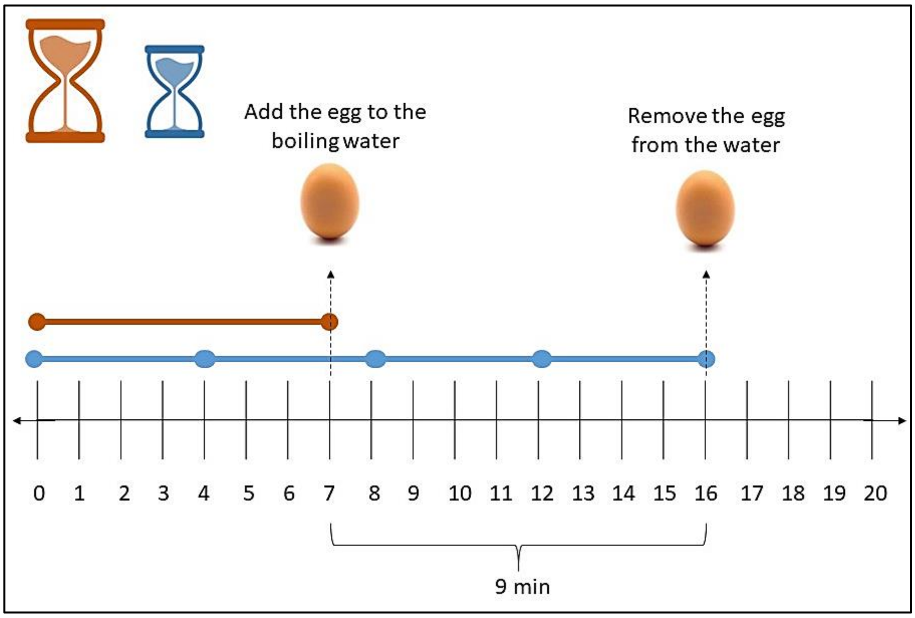

Figure 1: Possible solution to the Boiling Eggs problem

- *P2. Bar of the stressed people*: In the bar, there are 25 seats arranged in a row. All the customers who come to the bar are stressed, and each time they enter, they look at which of the 25 seats are available. If all the seats are empty, they sit anywhere, but if any are occupied, they sit at the maximum possible distance from the other customers. If the bartender could choose where to seat the first customer, where would it be best to seat them to accommodate the maximum possible number of customers? (adapted from Paenza, 2015).

This problem originates from recreational mathematics. In mathematical terms, it corresponds to a discrete optimisation problem that includes elements of combinatorial analysis. Students must develop a strategy to find the correct answer, which can be seat 9 or 17 (multiple solutions problem). Starting with either of these seats allows the maximum number of people to be seated (in the odd-numbered seats) without anyone sitting next to another person. Ultimately, the problem can also be solved through trial and error. Furthermore, the problem can also be solved using multiple representations, such as numerical representations or pictorial representations. Figure 2 offers an example of the representations developed by three groups of students while attempting to find the correct answer.

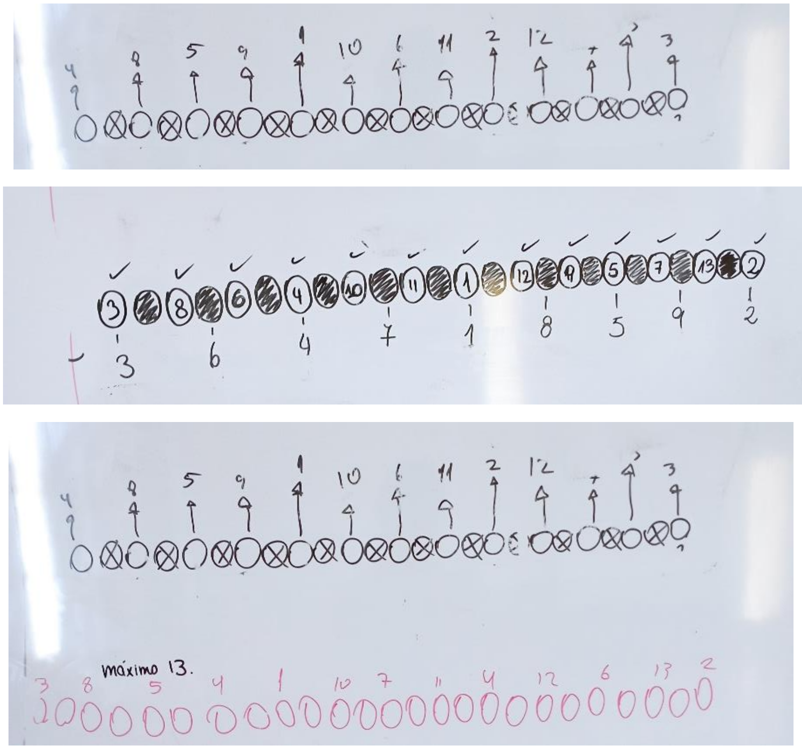

Figure 2: Students' representations while attempting to solve Problem 2

- *P3. Exploding dots:* Transform numbers from one base to another using the Exploding Dots representation (Tanton, 2009).

This is a curricular problem, as it was used as an introduction to the unit on converting numbers from the decimal system to other bases. Developed by James Tanton, the Exploding Dots system consists of a visual metaphor to explain the functioning of the decimal numbering system and other bases through the representation of an imaginary machine of dots that explode under certain rules. Each exercise has only one correct answer and a single mode of representation (pictorial). Figure 3 shows an example of a representation used by students attempting to solve the following problem: Convert the number 11101 from base 2 to base 10.

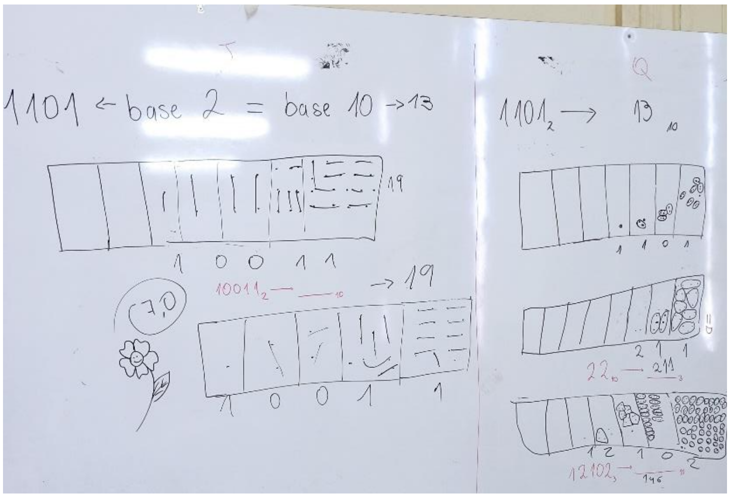

Figure 3: Students' representations while attempting to solve Problem 3

- *P4. Decomposition of 25:* Decompose 25 additively in different ways. Then, for each decomposition, multiply the obtained addends together. Which way produces the largest result? (Liljedahl, 2020).

This curricular problem was used as an introduction to the unit on operations with natural numbers in primary education. Solving it requires applying knowledge of additive decomposition and calculation of products, as well as knowledge of exponents, to arrive at the largest result. The problem allows for various solutions, depending on the attempts made by the students. For example, one group of students might suggest:

$$\text{Step 1: } 2+2+2+2+2+2+2+2+2+2+2+3=25$$

$$\text{Step 2: } 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2\cdot 3=6144$$

On the other hand, another group of students might answer:

$$\text{Step 1: } 3+3+3+3+3+3+3+2+2=25$$

$$\text{Step 2: } 3^7\cdot 2^2=8748$$

Depending on the students' skills, it is possible to arrive at a result that is the highest for them, but not the highest possible. At the same time, depending on their abilities, they will reach higher or lower levels of generalisation in finding the answer. This problem is mainly solved through a symbolic representation (mathematical language).

- *P5. The Creative Baker:* Find the fraction that represents each piece of a cake (Liljedahl, 2020).

This is a curricular problem, used as an introduction to the unit on fractions. The mathematical process required to solve it involves students recognising that a fraction consists of dividing a whole into equal parts, and then identifying which fraction (number) corresponds to each part. The difficulty lies in the fact that the pies are not necessarily divided into equal parts, so students need to find a way to make the parts equal in size. There may be only one solution, and one type of representation (pictorial).

- *P6. Field Trip:* 173 primary school students are going to Fray Jorge Park by bus. The park requires one adult supervisor for every 12 students. The bus company requires at least 2 adults on each bus. There are 6 teachers. The rest of the adults will need to be parent volunteers. Each bus has a capacity of 30 passengers. How will everyone be distributed? (adapted from Liljedahl, 2020).

This is a curricular problem. It was used as an introduction to a unit on multiples and divisors. To solve the problem, students have to calculate the necessary number of supervisors, estimate the number of buses needed, distribute students and supervisors on the buses, and calculate the number of parent volunteers required. This can be done using pictorial representations, tables, and purely mathematical symbols. Multiple correct solutions may be found.

A summary of the characterisation of the problems solved by the student teachers in each Thinking Classroom session is presented in Table 2 below.

Table 2: Characterization of the problems solved in each session

	Type of problem according to its origin	**Mathematical procedure**	**Number of possible solutions**	**Types of representations**
P1	Recreational	Logical reasoning	Multiple possible solutions	Multiple (pictorial, symbolic, etc.)
P2	Recreational	Maximisation	Two possible solutions	Multiple (pictorial, symbolic, etc.)
P3	Curricular	Number base conversion	One possible solution	Pictorial
P4	Curricular	Additive decomposition. Powers	Multiple possible solutions	Symbolic
P5	Curricular	Identifying Fractions	One possible solution	Pictorial
P6	Curricular	Logical reasoning. Maximisation.	Multiple possible solutions	Multiple (pictorial, symbolic, etc.)

5.2 Perceptions of the Solved Problems

Table 3 provides a summary of the results obtained in the first part of the questionnaire regarding the perception of different characteristics in the solved problems.

Table 3: Summary of results regarding the perception of problem characteristics

Problem	Difficulty	Interest generated	Contribution to learning	Enjoyment	Collaborative work
Decomposition of 25	2.44	2.63	3.40	2.88	3.07
Boiling eggs	4.57	4.03	3.92	3.96	4.38
Bar of the stressed people	4.14	3.51	3.00	3.37	3.37
Exploding dots	2.37	4.40	4.59	4.74	3.96
Field trip	3.26	3.69	3.34	3.26	3.61
The creative baker	4.40	3.11	3.11	2.92	3.11

As shown by the evaluation of the problems in terms of difficulty, interest generated, contribution to learning, enjoyment, and collaborative work, the problems with high difficulty levels, such as Boiling Eggs and The Creative Baker, tend to involve a high level of interest and promote collaborative work. Moreover, in these two sessions, the students demonstrated a high level of concentration. The Boiling Eggs problem stands out because it was implemented during the first week of classes, when the students were just getting to know each other. Additionally, the students had only their prior knowledge from school to help them solve it. The data from the field log suggest that allowing multiple representations and possible solutions may have contributed to promoting collaborative work, as each student could offer a different point of view. In this regard, one of the students said to her classmates:

> *"We can solve it like this: this hand is the 4-minute timer, and this one is the 7-minute timer. If I flip it like this [hand gestures] and wait until it finishes, I can flip the other one like this [hand gesture]. But I would still be missing one minute."*

After seeing her gestures, another student drew a number line on the board:

> *"What if we do the same thing but don't put the egg in at the beginning, but rather after the second flip?"*

Similar dialogues were repeated in other groups, suggesting that open-ended problems allow for a greater alignment with the socio-constructivist approach.
In contrast, the Decomposition of 25 and Exploding Dots problems were perceived as being the easiest, indicating that students considered them less complex. In fact, for the Exploding Dots problem, the method's operation was explained at the beginning of the class, and the students only had to replicate it through a series of exercises. Regarding the Decomposition of 25 problem, the field log indicates that students still had little autonomy in seeking diverse

solutions. For example, it was common for dialogues such as the following to occur:

> Student: *"Teacher, is it okay if I do it this way?" [writes 24 + 1 on the board]*
> Teacher*: "There's no problem with trying it that way."*
> Student*: "And if I do it this other way, is it okay?" [writes 1+1+1+1+21]*
> Teacher*: "You can also do it that way; you should give it a try."*
> Student*: "And then I need to multiply this, right? Is it okay if I do it this way to find the result?"*
> Teacher*: "I can't give you the correct answer, but you should try and see which way gives you the largest result."*

The Exploding Dots problem was perceived as having contributed the most to learning, indicating that students found this problem particularly useful for reinforcing their knowledge. In contrast, the problems The Creative Baker and Bar of the Stressed People were considered to contribute the least to learning. This could be due to the nature or approach of the problem, or perhaps because they did not align with the students' prior knowledge. In this regard, a student remarked:

> *"Teacher, when I was in school, I didn't like fractions. That's why I don't understand much."*

In terms of enjoyment, the students most enjoyed solving the Exploding Dots problem, further reinforcing the notion that open-ended or interactive problems tend to be more engaging. On the other hand, the problems The Creative Baker and Decomposition of 25 generated the least enjoyment, suggesting that the students were not as motivated or engaged with these problems.

5.3 Perceptions of the Approach

Regarding the open-ended questions, Table 4 shows the categories that emerged concerning the question about the variables considered in Dimension 3.

Table 4: Summary of the results of the content analysis of Dimension 3

Variable	Emerging categories	*f*
Contribution to learning	Useful for learning and understanding content	11
	Forced to think differently, ponder, try, find solutions	6
	Memorability	3
	Intrigue, curiosity, interest in learning something new	2
	Teamwork	2
	Fun	2
	Motivation	1
The most difficult aspects	Group work and its difficulties	8
	Autonomy required, teacher not answering questions	4
	Thinking (reasoning), understanding how to solve a problem	4
	Mental block	3
	Uncertainty due to unfamiliarity with the solution method	3
	The type of problems, out of the ordinary. Understanding them	2
	Communicating mathematical ideas	1
	Continuing to think (extending the problem)	1
	Achieving concentration	1

What students liked most	Collaborative work, camaraderie	9
	Deep reasoning, thinking deeply	7
	Satisfaction of finding the result yourself	3
	Autonomy in thinking	3
	The challenge	2
	Activity: innovative, entertaining	2
	Problems could be solved in different ways	2
	Using the board	2
	Participation	1
Contribution compared to a traditional lesson	Innovative, entertaining, interactive	7
	Critical thinking and reasoning	7
	Practical and didactic learning	6
	Autonomy	5
	Creativity	2
	Active participation	2
	Confidence	2
	Collaborative work	2
	Motivation	1
Contribution to teacher training	Implements practical, interactive, and playful approaches beyond the monotonous	13
	Helps to motivate learning	6
	Ways to foster logical-mathematical thinking	3
	Future learning: understanding the purpose of the content	3
	Fosters autonomy	2
	Changes the way of seeing mathematics	2
	Use of the board	2
	Understanding how students feel	1
	Knowing how to approach collaborative work	1

In terms of the contribution to learning, the results indicate that most students perceived the problems as being useful for learning and understanding the content; many also appreciated the cognitive challenge they offered. Memorability and curiosity are also highlighted aspects, although to a lesser extent, along with fun and teamwork. These results indicate that the problems not only helped in the acquisition of knowledge but also fostered critical thinking skills, collaboration, and enjoyment of the learning process. Regarding the Exploding Dots problem, some students noted:

> *"Although at first I thought it was useless to learn, it can be a good activity where students discover the content that will be taught. In this case, it could be a good introduction to learning exponents."*
>
> *"The Exploding Dots problem helped me as a guide to move from one base to another in an interesting and creative way."*

Most students highlighted the group work and the required autonomy as being the most difficult aspects of learning through problem-solving, as well as reasoning and understanding the problems. This indicates that while the approach challenges students to be more independent and collaborative, it also reveals areas in which they could benefit from additional support. Improving teamwork skills, providing strategies for critical thinking and problem-solving, and offering guidance on how to overcome mental blocks and uncertainties could help to mitigate these difficulties and enhance the learning experience. Consequently, the teacher's role during the activity is crucial in overcoming these challenges. In relation to this variable, some students indicated:

> *"The most difficult part during these classes was conveying my ideas or thoughts to [the] other group members, and making sure these ideas were understandable to them."*
>
> *"For me, the hardest part was directly engaging with my classmates and demonstrating my ideas on the board. That is, it's not a type of approach where you can work individually without fear of peer criticism. I am a shy person who prefers to work alone, so the most challenging part was working with people from different backgrounds and with different training than mine."*
>
> *"The hardest part was merging the three opinions of the group, whether during the development or the explanation of the answer, to reach a single conclusion."*

In terms of what the students liked most about the problem-solving approach, students' preferences were focused on collaborative work, deep reasoning, and autonomy in thinking. This indicates that the students valued both social interaction and independence in problem-solving. Furthermore, they also appreciated the challenges and innovation involved in the activities, as well as the flexibility required in solving problems. These results suggest that the approach succeeded in engaging students meaningfully, fostering both collaborative learning and the development of personal skills. Regarding this variable, some students indicated:

> *"What I liked most was that it was very entertaining to solve the problems, and when I reached the result, I felt very good about myself."*
>
> *"What I liked most was the collaborative work and the freedom of thought that it promotes. Also, the freedom to choose which tools to use to solve the problems we were given. Additionally, I liked the autonomy we had in our work, because with minimal guidance from the teacher, each of us had to think and develop our own reasoning."*
>
> *"What I liked most about this approach was working with different classmates and getting to know what mathematical skills each one had. Moreover, most of the activities allowed for reasoning since the answers were not straightforward, as we are usually accustomed to."*

The students' perceptions of the approach suggest that, compared to a traditional lesson, it offers several key benefits, such as being innovative, entertaining and interactive, as well as fostering critical thinking, reasoning, and practical learning. Autonomy, creativity, active participation, confidence, and collaborative work were also valued, though to a lesser extent. Thus, these results suggest that the students consider this approach to be more engaging and effective for their learning, promoting important skills and enhancing their commitment and motivation for the educational process. Some students indicated:

> *"I think the contribution of this approach is that it motivates and encourages critical thinking rather than directly giving the answer to the student. This helps them become more creative and proactive, as well as encouraging collaborative work."*
>
> *"Compared to a traditional mathematics class, this approach is more interactive. It is not just teaching content after content, but making students discover the why and the how. This way, they are made to reflect in the process of finding the answers to the problems."*
>
> *"The contribution of this approach is working on logical reasoning exercises, unlike a typical math class where the focus is on solving exercises that have more direct solutions."*

Finally, most students perceived that the approach contributes to their teacher training by promoting a practical, interactive, and playful method, motivating learning and fostering logical-mathematical thinking. Additionally, it was noted that they valued understanding the purpose of the content and the fostering of autonomy. Others highlighted various aspects including transforming the perception of mathematics, effective use of the board, empathy towards students, and managing collaborative work. In summary, these results suggest that the approach is effective not only in teaching mathematical content but also in the comprehensive preparation of future teachers. In this regard, some students pointed out:

> *"As a future teacher, having experienced teaching through problem-solving with this approach allows me to have a repertoire of ideas and strategies to foster logical-mathematical thinking in my future students. This way, I can implement this subject with a more entertaining and meaningful approach for them."*
>
> *"I think it is a valuable tool to use in the classroom. Additionally, it is motivating for starting a class or a unit, and it helps to create interest in students. On the other hand, it is a key strategy for developing collaborative work."*

6. Discussion

The primary aim of this research was to explore the perceptions of future primary education teachers regarding the implementation of a problem-solving-based teaching approach from a socio-constructivist perspective. The results of the study provide valuable insights into how future teachers perceive the effectiveness of problem-solving approaches in their own training, which is essential to ensure their eventual adoption and use in classroom environments. Thus, these findings are key to addressing the research problem, as they shed light on the benefits and potential difficulties of integrating problem-solving approaches into teacher education programmes.

Several key aspects were revealed in the results that reinforce the socio-constructivist approach of the approach used. First, students' positive perceptions of the problem-solving approach highlight its effectiveness in developing critical skills such as logical thinking, collaboration, and autonomy, even when students encounter difficulties along the way. These perceptions align with the literature in supporting constructivist approaches to mathematics teaching, as noted by Lee et al. (2022), who argue that such approaches promote deeper and more meaningful learning by allowing students to actively engage with mathematical concepts.

In particular, Csikszentmihalyi's (2003) Flow Theory is reflected in the high enjoyment and motivation scores that students reported in relation to more open-ended and interactive problems such as the Boiling Eggs problem. Although the relationship between enjoyment and challenge is well-documented in the literature, this study serves to reinforce the idea that problems allowing multiple solutions generate greater cognitive engagement and enjoyment among students. In this regard, Ryan and Deci (2000) pointed out that intrinsic motivation, being deeply related to self-determination and a sense of achievement, is a crucial factor for success in educational contexts.

On the other hand, problems that were perceived as being less challenging or having a single solution path, such as Decomposition of 25, were those that contributed the least to learning and generated less enjoyment. This finding reinforces Schoenfeld's (1992, 2020, 2023) view on the importance of presenting problems that offer an intellectual rather than merely computational challenge, thereby allowing students to engage with the problem from a more reflective and creative perspective.

Finally, it is noteworthy that some students found the autonomy required by this approach challenging, which is also consistent with previous research. For example, Fukuzawa et al. (2017) emphasise that problem-based learning can present difficulties for students who are not accustomed to taking a more active role in their own learning. However, these challenges also highlight the opportunity for future teachers to develop essential skills such as collaborative problem-solving and independence in pedagogical decision-making, which are fundamental for their professional training.

Although this study offers valuable insights into the perceptions of future primary education teachers regarding a problem-solving-based teaching approach, it also opens up several avenues for future research. One area requiring further exploration is the long-term impact of such approaches on teacher practice and student outcomes. Longitudinal studies could track how these future teachers implement problem-solving approaches in their own classrooms and examine their sustained effects on student engagement, critical thinking, and achievement. Additionally, there is a need to investigate the specific challenges of fostering autonomy and collaborative work in problem-solving contexts, particularly in diverse student populations with varying levels of mathematical proficiency. Future studies could also explore how teacher support and scaffolding during the early stages of implementing these approaches affect students' confidence and effectiveness in problem-solving.

7. Conclusion and Recommendations

The objective of this study was to explore the perceptions of future primary education teachers regarding the implementation of a problem-solving-based teaching approach from a socio-constructivist perspective. The main findings indicate that students positively valued the approach, highlighting its ability to foster critical thinking, collaboration, and a deep understanding of mathematical concepts. However, some challenges were also identified, such as the difficulty of group work and the level of autonomy required in this approach. One of the key implications of this study is the need to integrate problem-solving-based approaches into teacher training programmes, in order to adequately prepare future teachers to face the challenges of 21st-century mathematics education. Additionally, the importance of promoting social interaction and intrinsic motivation in the learning process is highlighted, as supported by such theories as Csikszentmihalyi's Flow Theory and Ryan and Deci's Self-Determination Theory. A recommendation for futures studies is to replicate this research in different contexts and populations and to further investigate the challenges presented by the need for autonomy and collaborative work in problem-based learning. Additionally, it is recommended that teacher support be increased during the initial stages of implementing these approaches.

8. Limitations

Among the study's limitations is its focus on a single educational context, which restricts the generalisation of the results. Additionally, the study design relied primarily on self-reported perceptions from students, which may introduce biases. While qualitative data analysis techniques were suitable for this study, future research could benefit from more extensive quantitative analyses.

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