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Change in the Era of Common Core Standards: A Mathematics Teacher's Journey

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Abstract. This article describes the changing practice of a seventh grade mathematics teacher as she participates in a professional development program that focused on how students think about and solve problems involving rational numbers. Data sources included pre and post test data on both teacher content measures and measures of knowledge of student thinking, observations during professional development workshops, classroom observations and interviews with the teacher. The data showed that the most significant aspects of the professional development workshop to impact the teacher's change process were the classroom embedded workshops and her expanding knowledge of how her own students solved problems. The results provide an encouraging opportunity to effect changes in classroom practice of secondary mathematics teachers.

Keywords: Teacher Change, mathematics teaching, professional development

Introduction

The interplay between teacher knowledge and teacher change in mathematics classrooms has captivated researchers for decades. Throughout various transformations in mathematics education such as, changes in standards and approaches to curricula development and implementation, studies of changes in teachers' knowledge and beliefs about effective mathematics instruction have highlighted the influence of a multitude of factors that either enhance or limit their ability to become more effective in their practice (e.g. Goldsmith, Doerr, & Lewis, 2014). The newly adapted Common Core Standards for Mathematics (National governors association center for best practices, council of chief state school officers, 2010) by a majority of the states provides yet another impetus for mathematics teachers to change their practice for improved student understanding and achievement in mathematics.

This article describes the evolution of one middle school mathematics teacher's self -efficacy over a five year period that includes both her experiences as a preservice and inservice teacher. The purpose of documenting her change over time is to attempt to provide a framework for analysis of secondary mathematics teacher reform in light of overall changes in expectations for mathematics instruction (CCSS-M, 2011). This framework could provide direction for secondary mathematics professional development. Three areas of study of mathematics teachers are integrated in this framework for analyzing teacher change: Mathematical Knowledge for Teaching (Ball, et al., 2008), Teacher Knowledge of Student's Thinking (e.g. Carpenter, et al.,1989), and Teacher Efficacy (e.g. Archambault, Janosz, & Chouinard, 2012).

Ball (2008) described six sub-categories of mathematical knowledge for teaching. This article focuses on the pedagogical content knowledge sub-category, "knowledge of content and students" by describing a professional development (PD) program focused on students' mathematical thinking and learning trajectories that the classroom teacher, Mrs. C, participated in for three years. Her participation in this PD combined with her evolving self-efficacy throughout the three year program is described. In particular, two aspects of the PD placed Mrs. C in a state of disequilibrium that eventually led to her shift in beliefs that the students could be successful if she taught with an emphasis on problem posing and assessing student thinking.

Knowledge of content and students could be characterized in a variety of ways. In this case study, it is used specifically to describe teachers' knowledge of students' mathematical thinking processes and trajectories within specific mathematics content areas. The professional development program provided teachers with research based information about how students' responses to particular types of word problems can be anticipated and used as part of the decision-making process.

Mathematical Knowledge for Teaching

The distinction between content knowledge and pedagogical content knowledge provided a lens for researchers to more closely examine the specialized knowledge needed to be successful as a teacher. Ball (2010) characterized components of mathematics knowledge for teaching by further delineating aspects of both subject matter knowledge and pedagogical content knowledge. Ideally, mathematics teachers would have strengths in all six of these aspects. Socio-cultural theories have also been used to characterize teacher learning (Goos, 2013). Socio-cultural theories provide a lens through which to explore the constraints faced by teachers as they attempt to change their practice and adapt reformed based instructional methods.

It may appear self-evident that secondary certified mathematics would require less professional development for mathematics instruction because their subject-matter knowledge would already be in place. The literature in fact is replete with studies of the impact of professional development programs on elementary teachers' content knowledge of mathematics (e. g. Ball & Bass, 2000, etc). The overarching assumption is that elementary teachers would not generally have a comparable knowledge base as secondary certified teachers who commonly have undergraduate degrees in mathematics. Therefore many professional development programs for elementary certified teachers incorporate mathematics content components into the workshops/seminars. Secondary professional development programs for mathematics teachers tend to focus more on curricular and/or technological aspects of teaching middle or high school mathematics (e.g., Cheung & Slavin, 2013).

One potential limitation of these apparent distinct goals of professional development for elementary versus secondary certified teachers is the lack of consideration for the interaction between understanding mathematics and understanding students' thinking about mathematics (Nathan & Petrosino, 2003). Studies of elementary teacher professional development programs and measures of teacher content knowledge have incorporated students' approaches to solving problems in attempting to improve teaching and learning mathematics (Hill, Rowan, & Ball, 2005; Franke and Kazemi, 2001). Franke and Kazemi (2001) discussed the idea of "generative growth" with respect to teachers learning about their students' approaches and progressions in solving mathematics problems (p. 105). In other words, the dynamic nature of student thinking about mathematics provides a mechanism for teachers to learn more about mathematics by engaging in the process of analyzing their own students' approaches.

Much less is known about risk-taking on the part of secondary teachers who are willing to base their instructional decision-making on their students' mathematical strategies. In particular, secondary mathematics teachers face a variety of hurdles in attempting to change their instructional practice (Daun-Barnett & John, 2012). Their challenges are multi-faceted. Curricular constraints such as textbook materials, pacing guides and standardized assessments typically prohibit student thinking approaches to instruction. Institutional factors such as scheduling constraints, administrative and peer pressure to conform, gaps in students' knowledge are just some of the factors that inhibit opportunities for teachers use new information or knowledge to change their practice.

Student Thinking Approaches to Professional Development

The knowledge base on children's problem solving approaches and levels of thinking in the area of whole number operations and algebraic reasoning is considered robust (Carpenter, et al., 1999; Fuson, 1992). The progression from representing all the quantities in the problem to more sophisticated strategies that utilize specific number relationships linked to number operations is well defined. This research base is considered robust in terms of the descriptive and comprehensive levels of student's thinking. This information became the basis of the well-known and studied professional development program entitled, "Cognitively Guided Instruction" or CGI. Several studies of the CGI professional development program documented the effects of teachers' increasingly detailed knowledge of students' understandings on their practice and "beliefs" about teaching mathematics (Fennema, et. al., 1993, 1996).

The longitudinal study of the effects of CGI professional development on teacher's knowledge and changing practice over a four year period showed that for most of the CGI teachers their practice continued to progress toward an emphasis on individual student's mathematical thinking and strategy progressions (Fennema, et al., 1996). The degree to which teachers utilized individual students' thinking as the basis for instructional decision-making was characterized by 5 levels of beliefs ranging from does not believe students can solve problems without instruction or that students are capable of using their own strategies (Level 1) to the belief that students are capable of solving problems on their own without pre-instruction on the topics and that knowledge about students' thinking should inform future instructional decision-making (Level 4a). The study showed that the majority of the teachers became more focused on student's thinking over the course of the four years of ongoing professional development.

Secondary Mathematics Professional Development Programs

Designing effective professional development for secondary mathematics teachers has been an ongoing challenge for educators. Often times, generic programs fail to provide the specifics that teachers need to implement them with their own students. Content specific and research based programs, particularly programs that are sustainable over time, tend to have more impact on student learning (Guskey & Yoon, 2009; Loucks-Horsley, 2003). Guskey & Yoon (2009), in their analysis state, "…the professional development program efforts that brought improvements in student learning focused principally on ideas gained through the involvement of outside experts " (p. 496). They also advocate for sustained professional development rather than limited or one day only workshops.

Effective professional development programs for middle school teachers can be particularly challenging from both the mathematics perspective and the pedagogical perspective. For middle school teachers who are elementary certified, the mathematics content is more challenging for them. For middle school teachers who are secondary certified, understanding how more complex topics like rational numbers and algebraic reasoning can be made accessible to middle school students is often times elusive. For the former category of teachers, students' thinking becomes a vehicle to learn content not otherwise understood. For the latter category of teachers, it is less clear how the impact of a student thinking professional development model would influence their instruction.

The premise of this article is that for Mrs. C, who falls into the latter category, a secondary certified mathematics teacher, teaching seventh grade, related factors impacted changes in her self-efficacy toward teaching mathematics as a result of participating in workshops that focused on student's thinking. The combination of sustained professional development focused on students' thinking and approaches to solving middle grades mathematics problems, a classroom – embedded professional development component in which the frameworks of students' thinking and problem solving methods were confirmed by students in "real time", and her own state of disequilibrium in instances where she acknowledged that she was unsure of the mathematics embedded in the student's work, factored into Mrs. C's transformation from opponent to proponent of teaching mathematics using problem posing and student responses as the primary organizing mechanisms of her lessons.

The workshops for which Mrs. C participated were extrapolated from basic principles of CGI workshops (Carpenter, et al, 1999). Like CGI, this professional development program focused almost exclusively on student thinking in the content areas of fractions, proportional reasoning, and algebra (Empson & Levi, 2011; Carpenter, Franke, & Levi, 2001). The underlying basis for CGI professional development was that providing teachers with detailed information about how students solve problems and think about concepts of whole numbers and operations would improve their ability to plan and implement instruction that productively built off their strategies (Carpenter, et al, 1989; Fennema, et al., 1993). The core of CGI professional development is the attention to frameworks of student thinking in relation to problem type structure involving whole number ideas such as place value and properties of operations.

The combination of the growing knowledge base on students' thinking about fractions and proportions (e.g. Empson & Levi, 2011; Lamon, 2012) and the renewed calls for improved student performance in prerequisite algebra skills led to the creation of a professional development program that would later be referred to as *Thinking Mathematically in the Middle Grades* or TM. Primary elements of CGI professional development were extrapolated to TM workshops. For example, teachers were given the opportunity to explore problem type structure for fraction problems similar to analysis of whole number problem type structures in the CGI workshops.

The initial frameworks for study in the TM professional development workshops focused on students' approaches to solving *multiple groups* problems. *Multiple groups* problems are characterized as multiplication or division problems in which the amount of groups is a whole number and the amount in each group has a fractional amount (Empson & Levi, 2011). From a research and developmental trajectory perspective, the first of the problem types explored by the participants are equal sharing problems for the purpose of generating fractional quantities. Equal sharing problems have been well-researched across multiple grade levels as being robust problems for generating concepts of fractions as quantities and fraction equivalence (Empson & Levi, 2011). The framework of strategies for equal sharing problems includes making the distinction between coordinating the number of objects with the number of sharers and more random partitions of the objects such as repeated halving. Other strategy distinctions include additive, ratio, and multiplicative (Empson & Levi, 2011). Similar to CGI workshops, TM workshops are designed to engage teachers in a deep exploration of these strategy levels and what the levels represent in terms of students' understanding of the content.

Multiple groups multiplication problems and division problems are then explored as a way to consolidate and further extend students' understanding of fractions as quantities. Five consecutive days of the workshop are devoted to helping teachers gain a thorough understanding of these three basic problem types. Teachers are positioned as their own students in the workshop and are encouraged to solve these problems in ways that they think their students would solve them without formal instruction. They sort strategies by level of sophistication and reconstruct the strategy frameworks for the problems types. Teachers interview students as well as watch videos of students solving these problems in order to reinforce features that characterize different strategy levels. Teachers are encouraged to pose these problem types to their students without providing formal methods to them first.

Knowledge of Students' Thinking and Teacher Efficacy

Efficacy in relation to teaching is generally described as "the teacher's belief or conviction that he or she can influence how well students learn, even those who may be difficult or unmotivated" (Guskey, p. 41, 1987). Teachers with high self-efficacy believe that they can positively influence student learning. In Guskey's (1987) study of context variables that influence measures of efficacy, teachers of all subject areas in the study were more likely to accept responsibility for poor performance by students if it was entire groups or classes of students than for individual students. Within traditional models of mathematics instruction in which teachers are focused on showing students procedures and problem solving methods, they would be much more reflective on their methods or procedures that they taught that were not shown to be effective with their students as a whole group rather than an indication that they had not attended to the learning of individual students. Both the CGI and TM professional development programs focus on strategies that individual students use to solve problems.

The emphasis on individual students' thinking about different problem types without direct instruction positions teachers differently during instruction. For many teachers, it is counterintuitive to the view that students need procedural directions on how to solve problems prior to being given the opportunity to solve them on their own. Particularly in secondary settings in which instruction is more teacher centered, knowledge of content and teaching and knowledge of curriculum are more likely emphasized than knowledge of content and students (Ball, 2008).

One aspect of the reform movement in mathematics in general is the shift from teacher centered to student centered lessons. Teacher efficacy is potentially influenced by this shift. For example, student centered lessons might involve some type of investigation in which students are working individually or in small groups. Teachers would potentially transition from their own explanations and strategies to the work of their students. In their study of teachers' beliefs about mathematics reform, Collins and Gerber (2001) found that teachers' personal self-efficacy and outcome expectancy were influenced by student learning characteristics. "Consistently, teachers reported relatively low personal efficacy and outcome expectancy when confronted with scenarios in which students exhibited characteristics associated with LD such as poor strategy use and poor affect" (p. 67). This is also a typical finding with teachers early on in the TM professional development workshops. For example, many teachers are surprised to learn that most students initially solve a variety of fraction problems using a semantic drawing of the problem and representing all of the quantities in the problem. Comments such as, "I don't think my students would do the problem this way", or "I never would have thought to solve this problem this way", are typical from teachers early on in the PD.

Thinking Mathematically Professional Development Program

This professional development program was designed to help teachers of upper elementary and middle school students understand how these students think about and solve a variety of fraction and proportion problems. The workshop was part of a three year grant developed to improve teachers' content knowledge in rational number concepts and algebraic reasoning. The PD consisted of eight days of summer workshops with three follow-up workshops during the school year, at least one of which was a classroom embedded workshop held in a participating teacher's classroom.

The focus of the workshops was helping teachers understand how students respond to fraction problems without first giving instruction on formal fraction content. Teachers are asked to solve equal sharing problems as a student in the elementary or middle grades might solve the problem (Empson & Levi, 2011). For example, a problem like 2 cakes shared equally among 3 children might elicit the following response from teachers: "My students would say that each child gets a half of a cake and gives the leftover to the teacher". Teachers learn that within the framework of students' strategies for equal sharing problems, that that response would be characterized as "non-anticipatory" because the child did not coordinate partitions of each cake with the number of sharers.

The overall goal of the PD is to help teachers make sense of the research base on how students solve specific types of fraction problems and how initial context-dependent strategies link to more efficient and mathematically sophisticated methods. Part of studying how students solve fraction problems requires teachers to grapple with their own concepts and potential misconceptions related to fraction content. One of the classic examples of this is the "invert and multiply" algorithm for dividing by a fraction. Teachers and students alike struggle with why the algorithm works (e.g. Tchoshanov, 2011). By allowing students to solve multiple groups division problems in which the amount in each group is a fraction amount in ways that make sense to them, many students use strategies that intuitively apply properties of inverses to solve the problem.

Within Case Analysis

Case study methodologies provide a lens to study the details of one particular situation and/or individual (Yin, 2013). The case of Mrs. C, while not entirely unique from other participants in the program, is described in order to illustrate the influence of each phase of the workshop. This particular case study provided a structure for integrating the data sources linked to her changing efficacy and instructional practices over the five year period. Figure 1 summarizes factors that influenced Mrs. C's changing instructional practice.

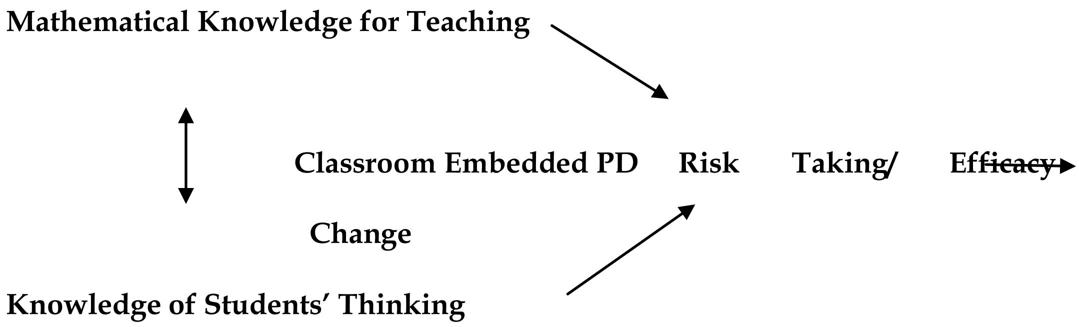


Figure 1. Factors that influenced Mrs. C's changing instructional practice.

Four data sources provided evidence of Mrs. C's changing practice over a five year period. Results of her performance on two teacher content measures, observations of her teaching, observations related to her participation in the TM workshops, and one-on- one interviews with her throughout the five year period were analyzed. Her case, while not entirely unique from other participants in the program, illustrates the influence of each phase of the workshop in Mrs. C's evolving personal efficacy over a four year period of time.

Mrs. C was one of 38 participants in the project. The participants included teachers and coaches spanning grades 4 through 8. Of the 38 participants, 23 were either elementary certified teachers or coaches. Mrs. C was one of 14 participants who were middle school mathematics teachers and one was a middle level special education teacher. The participants were from six school districts from a southern state. The districts represented both rural and suburban populations. The population of students at Mrs. C's school were approximately 75% minority (Latino and Marshallese students) and 90% free or reduced lunch.

Mrs. C's Background Experiences

Mrs. C participated in a secondary mathematics Master of Arts in Teaching program which included a year-long internship in three secondary school settings. She had a Bachelor of Science degree in mathematics with strong preparation in mathematics courses that included the Calculus sequence, Abstract Algebra and Geometry (look up her course work). Her work during her internship was considered impressive by her three different mentors and administrators. She was hired by the school in which she completed her middle school rotation prior to completing the MAT program. Observations of her teaching over the course of the year indicated that she had good communication skills related to mathematics content and good classroom management capabilities. In a traditional sense, she was considered a strong mathematics teacher.

As part of her mathematics methods experience in the MAT, Mrs. C was required to analyze the problem types and strategy levels for ratios and proportions (Lamon, 2012). She was then required to construct a set of problems and interview a student to assess her understanding of proportions based on the responses and strategies for solving the proportion problems. She was also asked to speculate on how the results of the interview might be useful in her instruction of ratio and proportion content. Like most of her peers, Mrs. C acknowledged that the information could potentially help her think various ways to introduce the content to students. In other words, she would have more methods that she could "show" students for solving proportion problems. She did not recognize that most of the problem types in the framework, with careful consideration to number combinations could be done by most students without teachers having to show them anything.

Following her first year as a seventh grade teacher, Mrs. C was nominated from her school district to participate in the three year TM professional development program. Two measures were used to assess teacher content knowledge. The first was the number and computation test DTAMS from Louisville (Saderholm, et al., 2010). The format of this test is multiple choice and short response. This instrument primarily assessed teachers' content knowledge of rational number content and operations with some attention to pedagogical content knowledge. The following problem is an example of a question from one of the versions of the number and computation tests:

Explain or demonstrate one way to help students understand
why 3/4 ÷ 2/3 = 1 1/8 other than teaching a numerical
procedure/process and observing that it results in this answer.
(University of Louisville Center for Research in Mathematics
and Science Teacher Development, Number-Computation, Version 6)

A variety of explanations could be used to explain or demonstrate this to students. However, the basis for the question is such that the teacher should respond on how he/she might explain the question to students as opposed to how students might determine the answer using their own methods. These tests were scored by the University of Louisville Center for Research in Mathematics and Science Teacher Development. Table 1 shows the comparison between the mean score of the grant participants on each of the tests and Mrs. C's scores.

Table 1
Mean scores on DTAMS Number/Computation measures

(total pts possible – 40)	Pre-test	First – post test	Second post test	Final post-test
Mean	25.8	26.0	26.6	32
Mrs. C	37	36	32	39

The second instrument was an assessment designed to specifically address teachers' knowledge of students' thinking about fraction and proportion content.

The Fraction and Proportion Thinking Inventory (FPTI) and rubric assessed teachers' knowledge of student approaches to solving various problems (Kent, 2009). The items were piloted with elementary and middle school teachers. The rubric was also revised based on the results of the field test. Additionally, the FPTI inventories were scored by members of the project until 90% inter-rater reliability was reached.

The questions on the FPTI instrument asked participants to anticipate how students at their grade level or students with general understanding of the topic would solve the given problems. Since the focus of the professional development was on presenting research on how students solve fraction problems without first having been shown a method or standard algorithm, the FPTI instrument was used to determine if teachers recognized these alternative approaches as possible ways that their own students might solve the problems. For example, question 3 asked the participants to show some ways that upper elementary and middle level students might solve the following problem:

A farmer has 15 ½ acres of land. If he divides the land in ¾ parcels,
How many parcels of land does he have?

As part of the TM professional development program, participants learn about students' strategies for solving measurement division problems involving fractional amounts. This problem is further defined as a *partial groups* problem because it involves a fractional group in the answer (Empson & Levi, 2011). They learn that students could solve the problem by representing all of the acres and all of the parcels to solve the problem or that they might use more multiplicative or relational strategies to solving the problem. A summary of possible strategies is given in Table 2. All participants completed the assessment four times throughout the three-year project: as a pre-test prior to the start of the professional development workshop, at the end of the first summer workshop and then at the end of the second year and third year respectively, of the workshop. Table 3 summarizes the mean scores of all participants over the three year project and shows the comparison between Mrs. C's scores and the mean scores for the participants as a whole.

As evidenced by high scores on both instruments relative to the participant group's mean scores, Mrs. C demonstrated strong content, pedagogical content knowledge, and knowledge of student's thinking about and approaches to solving problems related to fraction and proportion topics. Interestingly, her scores on both instruments dipped from year 1 to year 2 of the project but increased from year 2 to year 3 of the project which was also the same time that Mrs. C began to pose problems to her students on a more ongoing basis.

Changing Personal Efficacy

Mrs. C was one of 15 participants who attended all workshops all three years of the project. She also was one of four secondary certified mathematics teachers who participated all three years of the project. Her case is not dissimilar to the other three secondary certified participants. All four changed their practice over the three years to some degree to include more problem posing in their mathematics lessons. However, Mrs. C's particular case was instrumental in describing the phases that are potentially necessary for secondary certified teachers to transform their instructional practices from traditional, teacher-centered to student-centered, inquiry-based lessons.

Table 2
Possible strategies for the parcel problem.

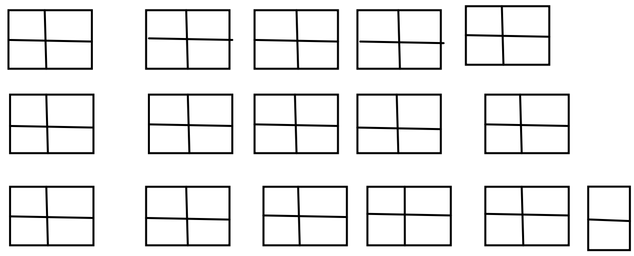
Strategy Level	Example
Represents all/additive	
Grouping/transitional	4 x ¾ = 3 8 x ¾ = 6 16 x ¾ = 12 20 x ¾ = 15 20 parcels with a ½ acre leftover
Multiplicative	Each acre contains 1 1/3 parcels, so 15 ½ acres x 1 1/3 parcels/acre = 20 2/3parcels

Table 3
Mean Scores of FPTI Assessment

(total pts possible – 16)	Pre-test	First – post test	Second post test	Final post-test
Mean	8.1	8.4	11.3	12.8
Mrs. C	12	14	13	16

Results: Change Process

Figure 2 shows the data sources used to analyze Mrs. C's change process over the five year period of her participation in the teacher certification program and for her first four years of teaching seventh grade mathematics.

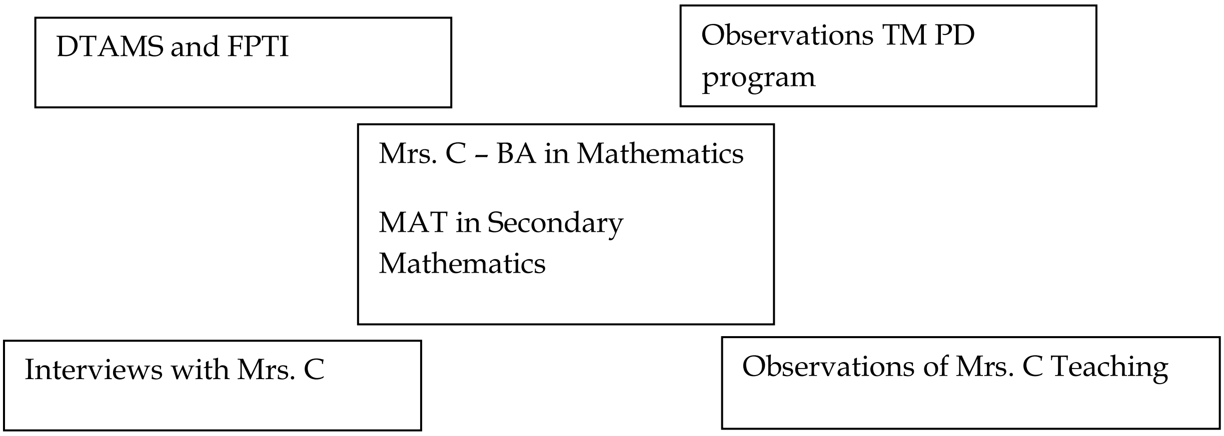


Figure 2. **Data sources analyzed for the case study of Mrs. C**

Interviews with Mrs. C throughout the course of the project also provide evidence of her changing personal efficacy. During her first year of teaching, prior to her participation on the project, Mrs. C acknowledged, "I basically told the students how to do the problems. I know the mathematics and it is my job to show them the correct methods". During her first year of participation in the project, she stated that she only posed problems to students when assigned to bring samples of student work to the seminar workshops. She further stated that most of the time she would assign her substitute to pose the problems to students, evidence that she did not consider it to be an important aspect of her own role to observe students as they attempted to solve the problems posed. She also commented that she did not feel that videos that were shown in the seminar style workshops were representative of her own students who were mostly minority population.

Following the second summer of seminar style workshops, the facilitators of the workshops, adapted a lesson study type of workshop protocol, entitled "Classroom Embedded" or CE workshop (Teachers Development Group, 2010) to use during follow-up workshops. Volunteers from the workshop were solicited to serve as host teachers for these workshops. The host teacher is responsible for teaching the lesson while the other participating teachers assist in the planning of the lesson and observe the implementation of the lesson. A sixth grade teacher from another school but within the same district as Mrs. C, volunteered to be the first host teacher for the CE workshop.

Prior to the classroom embedded workshop, the host teacher poses a problem to her students and collects the student work. The participating teachers sort the student work and determine a learning goal for the students based on their strategies for the previous problem and write a new problem/activity that they then observe the host teacher implement. They may also participate in choosing students to share their strategies and types of questions the host teacher would ask to build connections among key mathematical ideas.

Following this observation, Mrs. C began to pose problems to her lowest performing class of students, of which more than half were considered special education students. She did not use methods from the workshop with her more advanced students. She acknowledged in later interviews that "she wanted to prove that the ideas did not work" and that was why she chose that particular type of class to try problem posing. However, contrary to her initial hesitations about the methods, she noticed that these students began to show signs of problem solving ability. She stated that they began to show more willingness to persevere in solving problems. Their achievement as measured by state standardized tests, showed improvement, both from her previous year's students and from their own scores on previous tests.

The third summer of professional development included a component in which teachers had the opportunity to observe the facilitator of the workshop teach a fraction lesson to a small group of students. The goal was to provide an example of problem posing and eliciting student thinking. One of the key aspects of this lesson for Mrs. C was a reflection by the facilitator with respect to a student who had used a strategy that led to an incorrect answer. The facilitator stated, "I did not correct Isaiah because my goal was to understand his thinking".

At the end of this particular summer workshop, volunteers from the workshop were solicited to serve as host teachers for the CE workshops for the upcoming year. This time Mrs. C volunteered to serve as host teacher. She acknowledged that both the observation in the sixth grade teachers' classroom and the observation in the summer workshop validated her initial successes with the methods and gave her the confidence to implement on a more regular basis with her own students.

Opportunities for Risk-taking

The parallel between risk-taking that students engage in as they attempt to solve a novel problem for the first time is not unlike the risk-taking on the part of teachers as they attempt to change their teaching for the first time. The first standard, and potentially the most important standard for mathematical practice, "Make sense of problems and persevere in solving them" requires students to interpret mathematical situations and use their knowledge to determine strategies that will be productive towards a solution process (CCSSM, 2010). In a similar fashion, teachers using student thinking to drive instruction, must use their problem solving skills to make sense of their strategies in real time and decide on productive applications of their work to help students connect to big ideas of mathematics. The view of teaching as problem solving (Carpenter, 1989) encapsulates the complexities of teachers and classrooms and enhances teachers' sense of professionalism and autonomy with their own instruction. It empowers teachers as best positioned to make instructional decisions related to the mathematical learning needs of their students (Jacobs, et al., 2010).

Mrs. C began to increase and utilize her professional noticing of her own students as her knowledge of students' thinking increased and as she participated in classroom embedded workshops. Neither of these experiences in and of themselves would likely have changed her teaching practice. She acknowledged that she did not find the seminar workshops compelling in changing her thinking about instructional strategies. Observations of lessons without the structure of the classroom embedded protocol would not have given her the opportunities to make sense of the frameworks of student thinking applicable to her own students' strategies and therefore probably would not have prompted her to change. It was the intersection of these two experiences that provided the impetus for her to pose problems to her students and allow for their diverse methods.

Conclusion

This case study explores the changing practice of one middle school mathematics teacher as she engaged in professional development focused on students' mathematical thinking and learning trajectories. Even though students' thinking was a part of her graduate degree program, she did not adapt teaching strategies that allowed her to assess and build instruction on students' thinking until she observed the approach in another teacher's classroom with students she deemed similar to her own. The power of a *lesson study* style professional development experience was integral to her changing perception of her own students (Loucks-Horsley, et al., 2003). Mrs. C had a strong mathematics content preparation program, which is similar to most secondary mathematics majors. However, mathematics preparation is not the same as preparation in "Knowing the content and students" (Ball, et al., 2008). Measures such as the Fraction and Proportion Inventory (Kent, 2009) provide information about mathematics teachers' understandings of how students approach solving problems which are likely to include methods that are different from the teacher.

The case study of Mrs. C, a secondary certified teacher, provides information on her changing self-efficacy toward her students and her teaching practice as a result of a professional development program focused on students' thinking in specific mathematics content domains. One limitation was that the descriptions of the other teachers in the PD program were not detailed because most of them did not volunteer to serve as host teachers for the CE workshops. Giving all teachers the opportunity to have their teaching practices observed by their peers would determine whether or not this opportunity would change their instructional practices in the ways that Mrs. C changed her approaches to teaching mathematics.

Discussion

Mrs. C is not unlike many secondary certified teachers. She entered the teaching profession with degrees in mathematics and in education. Her collegiate experience was primarily received in lecture based classes and some attention to the role of student thinking within instructional decision making. Her field experiences, by all accounts, were traditional with the exception of utilizing technology resources such as graphing calculators, smart boards, and clickers, to facilitate instruction. Her case study exemplifies the complexities in attempting to capture the factor or factors that transformed her instruction over time. Three factors proved necessary in her change process: ongoing professional development focused on students ' thinking in specific domains, observations of the professional development model in "real time' with students deemed to be similar to her own students, and her changing self-efficacy concerning her impact on students.

This case study illustrates the potential for considerable changes in secondary teachers' classroom practices. In the era of Common Core standards, it is imperative for teachers to consider the accessibility of content for all of their students. Professional development focused on student thinking shows promise because it helps teachers understand how diverse learners make sense of problems in a variety of domains and therefore can enhance their options for moving their students toward understandings of important mathematics content. However, much additional research both on students' thinking in various secondary mathematical content areas and the potential influences of targeted professional development programs is needed. Other teachers from this professional development program began to change in a manner consistent with Mrs. C but were not systematically studied. These teachers and others like them need to be studied in order to determine if additional factors influenced their changes. Particularly, institutional supports should be explored in order to describe ways in which sustained growth can be encouraged beyond the span of the structure of the professional development programs.

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