

A Brief Review of Researches on the Use of Graphing Calculator in Mathematics Classrooms*

Jung-Chih Chen and Yung-Ling Lai
 National Chiayi University
 Chiayi, Taiwan

Abstract The main purpose of this study is to know about the graphing calculator used for learning mathematics. Many studies have attempted to find the effect of graphing calculator availability in mathematics classroom. Also, many educators have explored the role of graphing calculators. Based on reviews, most researches about the use of graphing calculators in the mathematics classrooms indicate that graphing calculators have had positive effect on the learning of mathematics at any grade levels. However, several reports still revealed that using graphing calculators had no significant or negative effects on learning of mathematics (Averbeck, 2001; Fox, 1998; Norris, 1995; Upshaw, 1994). This paper may provide many readers a snapshot to capture the use of graphing calculators based on research studies.

Keywords: Graphing Calculator, NCTM, CAS, Mathematics Classroom, Mathematics Learning.

Introduction

In 2000, the National Council of Teachers of Mathematics (NCTM) published the *Principles and Standards for School Mathematics*, a set of recommendations for mathematics curricula for Grade K-12. This book states: "Technology is essential in teaching and learning mathematics, it influences the mathematics that is taught and enhances students' learning" (p. 24). Also, the *Curriculum and Evaluation Standards for School Mathematics* document (NCTM, 1989) states that "The K-4 curriculum should make appropriate and ongoing use of calculators and computers" (p. 19). Thus, technology use in the classroom should be

*This study was partly supported by the Ministry of Science and Technology (MOST) in Taiwan. (Grant number: MOST 103-2511-S-415-003). Yet the opinions expressed here are only those of the authors.

**We are grateful for the helpful comments provided by several anonymous reviewers on earlier drafts of this article.

emphasized. Meanwhile, all levels of mathematics teachers should think about its impact and advantages to students' learning in the classroom.

In the spring of 2015, the researcher conducted a project to explore the implementation of mathematics content standards in high school in Taiwan, surveys were mailed to mathematics teachers. Of the 480 surveys, about 171 were returned. One item teachers were asked about software they used in Mathematics class. From the analyses in Figure 1 below, we found that most teachers only used MS Excel, clearly technology use was limited.

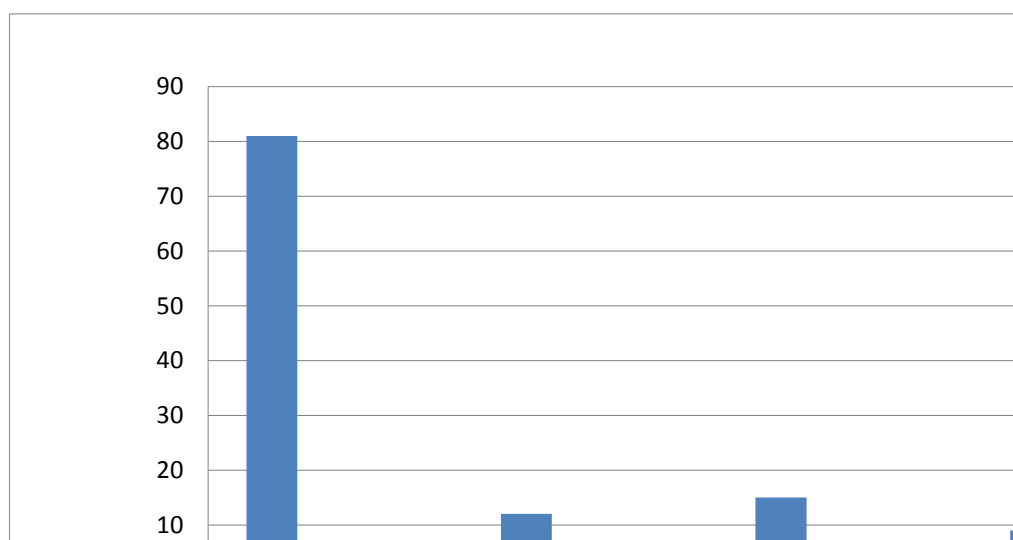


Figure 1: Responses to "What kind of software did you use in mathematics class?"

In the past, students usually only use paper and pencil in the mathematics classroom. But today most people agree that technology can engage students' interest in the learning process of mathematics. For instance, the reality is that calculator is able to perform operations and execute algebraic symbolic manipulations quickly and accurately; calculator use allows students and teachers more time to develop mathematical understanding, reasoning and other high level of applications. The fact is that appropriate use of technology associated with appropriate pedagogy will lead students to think and reason more mathematically.

This review focuses on the use of graphing calculators. As you may know, since the mid-1980s, there has been growing interest and attention in graphing calculators' potential to facilitate and enhance the teaching and learning of mathematics in school. From algebra I to pre-calculus, most studies designed have compared test scores between two groups; one called experiment group in which students received instruction with graphing calculators, and the other called control group received instruction without a graphing calculator. The final results of studies mostly suggest that the use of graphing calculators in teaching and learning is quite helpful to students' cognitive understanding, visualization, and achievement in mathematics classrooms. (Graham & Thomas, 2000; Johnson, 1997; Karadeniz, 2015; Kastberg & Leatham, 2005; Paschal, 1995; Wareham, 2016).

Academic Research

In this section, I have reviewed many researches related to the use of graphing calculators, especially focusing on some academic dissertations. Although about fifty dissertations have been carefully reviewed, because of the limitation of space, here I briefly discuss some significant exemplars as follows:

1. Blozy (2002) conducted his study to analyze performance on calculus questions by students using computer algebra system (CAS) and non-CAS groups respectively. In addition, students were given two calculus tests, but students were only allowed to use graphing calculator on the first test. Further, Blozy used both quantitative and qualitative methods to analyze the collected data.

Fifty-six advanced Placement high school calculus students participated in the study. CAS group included thirty students and non-CAS group included twenty-six students. All students were given two calculus tests, but students were only allowed to use graphing calculator on the first test.

Indeed, Blozy found that the CAS students performed better on some individual questions while non-CAS students performed better on the other individual questions. Overall, the results did show that CAS students and non-CAS students approach and answer questions differently. Specifically, the CAS students consistently used graphical and numerical representations to solve problems and seemed to perform better on questions requiring this type of representation. Likewise, non-CAS students consistently used algebraic approaches to solve problems and seemed to perform better on questions requiring these types of representations. This study concluded that it was not the type of question that was significant to performance, but the type of representation (That is; depending on its graphical, numerical, or algebraic type) that students used to solve the question that was significant to performance.

2. Averbek (2001) investigated college students' learning of the function concept and the role of the graphing calculator in a college algebra course. He also examined the difference between students with high symbolic manipulation skills and students' algebraic skills and academic majors (math & science, business, and liberal arts), twenty five students were involved in this study, and they were divided into six categories.

To collect data on students' understanding of functions, students are given a pretest and posttest. Some test questions consist of three problem situations given in the numerical, graphical, and symbolic representations. To collect data about the role of the graphing calculator, Averbek (2001) conducted daily classroom observations. Further, formal and informal interviews with students and instructor were conducted to verify students' responses and classroom observations.

The results indicate that students had difficulties with univalent requirement in three areas: (a) order of domain and range, (b) preference for simple algorithms, and (c) the restriction that functions were one-to-one. Students with high symbolic manipulation skills were more flexible working between representations of functions. Also, half of the students with low symbolic manipulation skills perceived a single function given in different representations as separate entities. Again, students might interpret that exponential functions possessed a bounded domain because they did not

explore the behavior of graph.

Averbeck also mentioned that the graphing calculator played a role in all phases of the solution process as follows:

- (1) The initial phase: Students used graphing calculators to develop a symbolic approach.
 - (2) The solution-execution phase: Students used graphing calculators to avoid careless errors.
 - (3) The solution-monitoring phase: Students used graphing calculators to check answers.
3. Allison(2000) investigated the challenge of the graphing calculator used in high school students' thinking while solving problems. The students were presented with many different tasks to solve including both contextual, non-routine problems and non-contextual, exploratory problems. With pertinent data presented in symbolic, tabular, and textual representations. Students interviews reflected their perspective on the importance of the use of the graphing calculators when problem solving.

The results reveal that the graphing calculator gives an impetus to a students' mathematical problem solving. All of the students agreed that the graphing calculator promoted speed and accuracy to their problem solving process. Frequently, students attempt to use graphical approaches to solve problems, and their thinking ways about tasks are also influenced.

Quite often, students might use the graphing calculator functions that had been demonstrated in their mathematics classroom. For instance; they used the "regression functions" to find symbolic models in problems with tabular data and in explorations with graphical data. Also they might use trial-and-error methods to explore the possible relationships between functions and their corresponding graphs.

4. Milou (1999) designed a survey of classroom usage involving the graphing calculator. In this study, he focused on 146 secondary mathematics teachers who used the graphing calculator in their teaching. In particular, main attention is to examine algebra teachers' teaching; their understanding about the graphing calculators, and to see any changes about their instructional practices.

Most teachers agreed or strongly agreed that "Graphing calculators allow for algebra classes to cover additional material." In addition, all teachers were asked to tell in which potential topics were appropriately explored. The following three topics were written by a lot of teachers:

- (1) Statistics associated with data analysis, curve or best fit, quartiles, and so on.
- (2) Some problems using complex applications involving in everyday life or business, something like that.
- (3) Mathematical problem solving.

Likewise, teachers were asked about what topics should be ignored or should be weakened; their answers mostly include

- (1) Factoring
- (2) Student produced graphing
- (3) Rational expressions

In addition, about 72% of the teachers answered "agree" or "strongly" to the

question: "Students should first solve algebraically and support graphically." But, when they were asked if students should solve graphically only when algebraic methods are unworkable, 59% answered "disagree" or "strongly disagree."

Other important results include (i) Most algebra teachers thought that the graphing calculator played a motivational tool, (ii) Teachers of algebra II used the graphing calculators more often than teachers of algebra I, (iii) Algebra teachers are not confident of how to use the graphing calculators in their teaching.

Milou also concludes that at present the use of the graphing calculator is not coincident to many algebra teachers. He points out the fact, although many high school teachers and algebra II teachers widely accepted the graphing calculator, yet there is still much controversy by middle school teachers and algebra I teachers, they express concerns about whether this technology is really appropriate in their mathematics classrooms.

Interestingly enough, two questions shown in this study are listed as follows: (Milou, 1999)

Q1: "Do you Use Graphing Calculators in Your Classroom?"

Current Teaching Assignment	Response(Yes)	Response(No)
Algebra I	37	45(teachers)
Algebra II	21	2(teachers)
Both Algebra I & Algebra II	19	3(teachers)
Neither Algebra I nor Algebra II	11	8(teachers)

Q2: "Should Graphing Calculators Be Permitted to Be Used on All Tests?"

Response	Frequency	Percent
Strongly agree	21	14.6
Agree	31	21.5
Neither agree nor disagree	20	13.9
Disagree	52	36.1
Strongly disagree	20	13.9

5. Fox (1998) studied the relations of a graphing calculator used in an active learning environment on intermediate algebra students' achievement and attitude. This quasi-experimental study involved six classes of community college intermediate algebra students (totally 166 persons) during six weeks. Students in the experimental group used TI-82 in class and on tests. While students in the control group still used their scientific calculators. Both groups were taught in active learning environments. Students were given pretest, three achievement tests and a post-attitude survey.

The results indicated that no significant differences due to treatment were found between the groups on achievements or attitude toward mathematics. However, when compared to intermediate algebra classes not involved in this study, 15% more students in the study's active learning classes completed the course successfully.

Fox also drew the following important conclusions in his studies:

- (1) Active learning could positively impact learning.
- (2) The use of graphing calculators encouraged active learning and could help students make connections between different representations.
- (3) Remedial students might not be accustomed to using calculators for studying mathematics and should be tested on the basic use of the graphing calculator before being tested for achievement.
- (4) Teachers might struggle to incorporate the use of graphing calculators.
- (5) The determination of technology's appropriate use should allow for the diversity of remedial students' needs and past experiences.

Highlights From Various Research

According to the research articles, some important findings about the use of graphing calculators could be succinctly summarized as follows:

- (1) The most critical personal factors affecting teachers' decisions were professional development, beliefs, mathematical and pedagogical knowledge, and familiarity with graphing calculators. Note that teaching experience, education background, and personal use of the calculator did not play an important role in teachers' decision-making process (Szombathelyi, 2001)
- (2) The graphing calculator was usually used as an exploring tool to solve problems related to limit and derivative. When a multiple representational approach was used, it was very likely to involve of algebraic and graphical representations (Girard, 2002).
- (3) Technology integration with the graphing calculator requires changes in teacher attitudes, course content, instructional methodology, and teacher preparation. Teachers with limited knowledge and preparation using graphing calculator usually exhibited a lack of confidence that affected their attitude and their effectiveness in using the graphing calculator in their classrooms (Bynum, 2002).
- (4) CAS students and non-CAS students approach and answer questions differently. The CAS students consistently used graphic and numerical representations to solve problems (Blozy, 2002).
- (5) Students used graphing calculators to develop a symbolic approach, to avoid careless errors, and to check answers (Averbeck, 20001).
- (6) The graphing calculators often can help students visualize the derivative and hence make connections with the other kind of representations (Serhan, 2000).
- (7) The results revealed that the graphing calculator served as impetus for students' mathematical problem solving. Graphing calculator could promote speed and accuracy to their problem solving process. (Allison, 2000).
- (8) The graphing calculators could enhance students' graph interpretation abilities. Students' abilities to interpret contextual graphs of functions both locally and globally improved. Students exhibited very well developed understanding of the relationship between slope and the rate of change of one variable (Pullano, 2000).
- (9) The graphing calculator proved to be useful tool for achieving an improvement in student understanding of variable. Some concerns include the lack of confidence and the lack of resources (Graham & Thomas, 2000).

- (10) Some algebra teachers are not confident of how to use the graphing calculator in their instruction. For example: Do concepts and procedures still need to be mastered first? Also, teachers agreed that the graphing calculator should be used to support analytical algebra findings, but they disagreed that the graphing calculator should only be used when algebraic methods were difficult. Teachers in the study believed that algebra I students were too dependent on the graphing calculator and were thus unable to handle basic algebraic manipulations (Milou, 1999).
- (11) The graphing calculator could make the work easier and sooner, it did enhance a student's visualization about Algebra II concepts. But, some students with low performance did have more difficulty in operating the graphing calculators (Drottar, 1998).
- (12) The cognitive benefits about graphing calculator use were still controversial by some algebra teachers. Most algebra teachers thought that graphing calculator was useful tool for promoting motivation in mathematics classrooms (Milou, 1998).
- (13) Some results suggested that the graphing calculator (i) could facilitate the learning of functions and the spatial visualization skills; (ii) could promote mathematical investigation and exploration; and (iii) could reform in emphasis of teaching and learning from algebraic skill to graphical investigation, and exploring the relationship between graphical, algebraic and geometric representations (Penglase & Arnold, 1996).
- (14) Teachers in high school used graphing calculators to provide graphs and make generalizations about transformations of the quadratic function in mathematics classroom. (Simmt, 1997).
- (15) Currence (1993) found that some teachers have changed the ways they taught mathematics because of the use of the graphing calculator in their classrooms.

Concluding Remarks

In summary, most studies conclude that using graphing calculator has its value in the mathematics classroom. At least, available research indicates that some benefits are obvious and straightforward, such as (1) It will provide students visualization (Karadeniz, 2015) through graphical and numerical approaches to solve problems, (2) It will reduce the time spent on calculations and manipulations as well, (3) It will illustrate some mathematical concepts which lead to a higher level of thinking and understanding, and (4) Results of an in-depth study reveal the positive relationships between calculator use and mathematics achievement (Kastberg & Leatham, 2005 ; Wareham, 2016). However, here we emphasize the importance of teacher's "appropriate use" in teaching. Teachers should look through the textbooks in advance to find topics where a graphing calculator could be used. Basically those topics should meet content goals and learning objectives of the students.

Further, some people may wonder if the graphing calculators have the possibility to reform mathematics education, such as the ways in teaching or the content focuses. In order for more significant changes to come through, here we provide some suggestions as follows:

- (1) Students , especially in high schools, should have access to graphing calculators to perform calculations and to support problem solving activities in mathematics classroom.
- (2) The school and district should support and guide teachers in the appropriate use of the graphing calculators. Professional development can provide in service workshops, peer tutoring, and new faculty expertise.
- (3) Since the graphing calculator becomes more popular by classroom teachers, so the curriculum needs to be reviewed and aligned, especially for some areas that can be de-emphasized because of the inclusion of the graphing calculators. On the other hand, in some topics, teachers can better have the opportunity to learn “when” and “how” to use the graphing calculator appropriately in their mathematics classrooms.
- (4) All publishers of textbooks, including all authors of assessments, evaluation, and mathematics instruments, should realize that the graphing calculator applications certainly affect the mathematics curriculum.
- (5) Those people who are responsible for the selection of curriculum materials should remain cognizant of how the technology eventually affects the curriculum.
- (6) Whether or not students are allowed to use the graphing calculators in their tests, we believe, is also an incentive factor to determine its prevalence in the mathematics classroom.

Finally, the researchers expect that mathematics teachers in high school can understand the benefits to integrate graphing calculators in their classrooms. Indeed, teachers can determine “when” and “how” to use it in class (Karadeniz, 2015). More than all, technology use should be considered in new curriculum documents such as mathematics content standards or guidelines.

References

- Alexander, M. P. (1993). The effective use of computers and graphing calculators in college algebra. *Dissertation Abstracts international*, 54(6A), 2080.
- Allison, J. A. (2000). High school students' problem solving with a graphing calculator. (University of Georgia, 2000). *Dissertation Abstracts international*, 61(11A), 4314.
- Arnold, S. M. (2003). Integrating Technology in the Senior School: Non-Calculus Mathematics. Texas Instruments Australia. Melbourne.
- Arnold, S. M. (2003). Integrating Technology in General Mathematics: NSW Stage 6. Texas Instruments Australia. Melbourne.
- Averbeck, P. J. (2001). Student understanding of functions and the use of the graphing calculator in a college algebra course. (Oregon State University, 2001). *Dissertation Abstracts international*, 61(11A), 4315.
- Blozy, T. A. (2002). An analysis of performance on calculus questions by students using CAS and non-CAS graphing calculators. (Columbia University Teachers College, 2002). *Dissertation Abstracts international*, 61(5A), 1754.
- Bynum, H. L. (2002). Graphing calculator use in college algebra and implications for teaching and course development. (North Carolina State University, 2002). *Dissertation Abstracts international*, 61(4A), 1281.
- Chandler, P. A. (1993). The effect of the graphing calculator on high school students mathematical achievement. *Dissertation Abstracts international*, 51(10), 3823A.
- Currence, A. J. (1993). A descriptive study of instructional delivery systems for

- mathematics teachers related to the graphing calculator. *Dissertation Abstracts international*, 53(11A), 3832.
- Devantier, A. T. (1993). The impact of graphing calculators on the understanding of functions and their graphs. *Master Abstracts International*, 31, 535.
- Drottar, J. F. (1998). An analysis of the effect of the graphing calculator on student performance in algebra II (High school students). Boston College, 1998. *Dissertation Abstracts international*, 61(1A), 56.
- Fox, L. F. (1998). The effect of a graphing calculator used in an active learning environment on intermediate algebra students' achievement and attitude. Ph.D. thesis, University of South Florida.
- Girard, N. R. (2002). Students' representational approaches to solving calculus problems: Examining the role of graphing calculators. (University of Pittsburgh, 2002). *Dissertation Abstracts international*, 61(10A), 3502.
- Graham, A. T. & Thomas, M. O. J. (2003). Building a versatile understanding of algebraic variables with a graphic calculator. *Educational Studies in Mathematics* 41: 265-282. Kluwer Academic Publishers. Printed in the Netherlands.
- Johnson, L. H. (1997). A look at parabolas with a graphing calculator. *The Mathematics Teacher*, 90(4), 278-282.
- Karadeniz, I. (2015). UCSMP Teachers' Perspectives when Using Graphing Calculators in Advanced Mathematics. *Graduate Theses and Dissertations*.
<http://scholarcommons.usf.edu/etd/5712>
- Kastberg, S., & Leatham, K. (2005). Research on graphing calculators at the secondary level: Implications for mathematics teacher education. *Contemporary Issues in Technology and Teacher Education* [Online serial], 5(1). Available: <http://www.citejournal.org/vol5/iss1/mathematics/article1.cfm>
- Milou, E. (1998). Attitudes toward and use of the graphing calculator in the teaching of algebra (high school teachers). (Temple University, 1998). *Dissertation Abstracts international*, 59(6A), 1956.
- Milou, E. (1999). The Graphing Calculator: A Survey of Classroom Usage. *School Science and Mathematics*, 99(3), 133-139.
- National Council of Teacher of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.
- National Council of Teacher of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author.
- Norris, C.W. (1995). The impact of using graphic calculators as an aid for the teaching and learning of precalculus in a university setting. *Dissertation Abstracts international*, 55(7A), 1862.
- Ottinger, T.P. (1994). Conceptual and procedural learning in the first year algebra using graphing calculators and computers. *Dissertation Abstracts international*. 54(8A), 2934.
- Paschal, S.G. (1995). Effects of a visualization-enhanced course in college algebra using graphing calculators and video tapes. *Dissertation Abstracts international*, 55(9A), 2754.
- Penglase, M. and Arnold, S. (1996). The graphics calculators in mathematics education: A critical review of recent research. *Mathematics Education Research Journal*, 8(1), 58-90.
- Penkow, C. K. (1995). The effect of college student use of graphics calculators on the learning of algebraic concepts. *Dissertation Abstracts international*. 55(12A), 3774.
- Pullano, F. B. (2000). Enhancing students' graph interpretation abilities through the use of graphing calculators. (University of Virginia, 2000). *Dissertation Abstracts international*, 61(6A), 2227.
- Serhan, D. (2000). The effect of using graphing calculations on students' concepts images of the derivative at a point. (Arizona State University, 2000). *Dissertation*

- Abstracts international*, 61(10A), 3932.
- Simmt, E. (1997). Graphing Calculators in High School Mathematics. *Journal of Computers in Mathematics and Science Teaching*, 16(2), 269-289. Charlottesville, VA: Association for the Advancement of Computing in Education (AACE). Retrieved January 17, 2016 from <http://www.editlib.org/p/8897>.
- Szombathelyi, A. (2001). Personal factors that influence teachers' decisions about graphing calculator in mathematics instruction. (University of South Carolina., 2001). *Dissertation Abstracts international*, 62(7A), 2368.
- Upshaw, J.T. (1994). The effect of the calculator-based graph-exploration method of instruction on advanced placement calculus achievement. *Dissertation Abstracts international*. 54(11A), 4023.
- Wareham, K. (2016). *Calculators and Mathematics Achievement: What the NAEP Mathematics Results Tell Us*. Paper presented on Jan. 3 at the (14th) 2016 Hawaii International Conference On Education. Honolulu, Hawaii, USA.