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Modelling in Vietnamese School Mathematics

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Abstract. This paper presents empirical research about implementing mathematical modelling in the secondary schools in Vietnam. The data from experiments have shown that there were some cognitive barriers in introducing modelling to the classroom and designing real world models for teaching. However, we have concluded that modelling is one of the active teaching methods and the teachers can provide their students with appropriate interventions to support them interpreting about the role of mathematics in reality. Furthermore, by participating in modelling activities, the students would develop their problem solving skills and regularly adjust their thinking.
Keywords. mathematical modelling; modelling teaching; modelling method; modelling process.

INTRODUCTION

In the last few decades, a lot of researchers have dealt with the problem of how to use mathematical modelling and its application in teaching mathematics at all levels in schools (Blum & Leiß, 2007; Maaß, 2007). In recent years, modelling has been considered as a new trend for research on problem solving in mathematics education (Lesh & Zawojewski, 2007; Kaiser, 2014). The results of this empirical research provided a new approach in teaching applied to mathematics and opened new ways of thinking about integrating real world situations in the process of learning and teaching school mathematics. Mathematical modelling is a process of applying mathematical concepts to new and unfamiliar situations. It relates to discovering a real situation, collecting data, making a hypothesis, building a model (equations, functions, symbolic structures, etc.), representing the model, interpreting the results, improving/revising the model, and answering the questions about real-world situations (Dan & Xie, 2011; Lingefjärd, 2006; Swetz & Hartler, 1991). A mathematical model is the product of the modelling process. It is created using suitable mathematical tools and methods. It can be expressed through a set of symbols, notations, graphs of data, geometric figures, tables, formulae, functions, equations, and systems of equations that describe complex relationships among real world situations or phenomena. Hence, mathematical modelling is an effective strategy to apply mathematics in interpreting various areas of life such as medicine, engineering, finance, economics, weather forecasting, ecology, sports, arts and computer science. More specifically, these phenomena can be described by laws of nature and mathematical modelling makes the laws predictable. For example, a graph of a parabola presents the motion of an object dropped from a height above the ground; a graph of exponential functions shows the population growth. Similarly, populations of predator and prey in an ecosystem, the unemployment rate, a risk factor for a disease, the effectiveness of a medical treatment, population growth, etc. can be represented by graphs of different functions that students are taught in schools (Biembengut & Hein, 2010; Blum & Ferri, 2009; Frejd & Bergsten, 2016). Hence, it could be said that mathematical modelling connects students' real life experiences with mathematics knowledge at schools. A modelling task engages students in solving a mathematically rich problem and developing mathematical thinking. It is a tool that helps students to understand about application of the mathematical concepts because it requires students to apply mathematical knowledge into real life and to extend the concepts beyond rote learning (Dan & Xie, 2011; Galbraith, Stillman & Brown, 2010; Lesh & Zawojewski, 2007; Kaiser & Stillman, 2015). Students should be provided an opportunity to build, determine and look for the best fitting model for real world data by sketching graphs of functions that represent important mathematical ideas and methods (NCTM, 2000). In the last few decades, curricula reforms in many developed countries have concentrated on mathematical modelling in their revised school curricula and textbooks.

Students are encouraged to look for situations in their real life and to pose the problems by making questions and formulating conjectures (Brown & Walter, 2005; Kang & Noh, 2012). Students need to understand the real world applications of mathematics so that they can solve problems both in everyday life and in the sciences. To solve mathematical modelling problems successfully, the teacher should teach their students how to do a realistic project, to collaborate with others and to create open discussion among members of the group. The teacher should also encourage students to use the functions of computers or calculators in modelling real life phenomena such as graphing tools, dynamic geometry environment, computer algebra system, simulations, dynamic spreadsheets, statistical packages, etc. In particular, during the teaching process, the teacher should encourage their students to use multi-representations of collected data (e.g., graphs, tables, equations, diagrams, pictures, etc.), and to choose and utilize appropriate mathematical methods and tools in solving non-routine problems.

In recent years, radical and comprehensive renovation of education and training at all levels has been a significant focus in Vietnam. This reform aims to meet the requirements of industrialization and modernization, dynamic systems of information in a knowledge-based economy, and international integration. In this renovation, there will be fundamental changes in curricula design and textbook compilation. The new curricula will be highly integrated in lower levels and highly selective and specialized in higher levels. By the end of the junior secondary education, students have acquired sufficient knowledge, virtue and necessary skills for vocational training and capability of new labourers. In the previous educational reform, modelling activities were under-emphasized in Vietnamese mathematics curricula and textbooks (Nguyen & Tran, 2013). However, the newly revised mathematics curricula have focused on important mathematical ideas and processes that promote students working with complex systems, such as investigating, conjecturing, justifying, representing, and explaining together with real life data and phenomena. As a result, modelling and problem solving have become core parts of the process of teaching mathematics at all educational levels. In other words, in new mathematics curricula, modelling and its application are important features. Students use modelling to get insight into mathematical concepts and understand about the applications of these concepts to students' life experiences. Mathematical modelling is also a compulsory competency within the national standards from primary to secondary school and students' competency profile.

THE MODELLING PROCESS IN THE CLASSROOM

Modelling as a Teaching Method

In this research, we consider modelling as a method of teaching mathematics aimed at providing students with an opportunity to read, to interpret, to formulate and to solve specific real world problems. This approach helps the students to use mathematical concepts to solve realistic problems that they may encounter in life. By using this method, students are able to look at the world through a mathematical lens and develop better comprehension of the world. Especially, as the students work with data from real life, they need to find mathematical ideas to understand the data and validate their conjectures. In primary education, teachers use figures, shapes, concrete materials, drawings, diagrams, and pictures to present arithmetic operations (e.g., addition, subtraction, multiplication, and division). In secondary education, teachers could guide students to use graphs and symbolic equations to represent relationships among quantities. In particular, in high school the students are taught about linear, quadratic, exponential, trigonometric, and polynomial functions as algebraic models. These models represent complex situations or phenomena around students' lives.

Throughout our study, we have concentrated on the feasibility of mathematical modelling activities in the classroom. We suggested that teachers provide their students with supporting questions or hints during tackling modelling tasks. In our point of view, this approach would train the following students' modelling skills: simplifying hypotheses; clarifying the goal; determining variables and parameters; formulating mathematical problems; selecting or building a mathematical model; graphical representations; and relating back to the real context. The context must be provided by the teachers. Therefore, the teachers need to look for real world situations or models and bring them to the classroom with the aim of helping their students to interpret the nature of a mathematical concept as well as its applications in real life. In other words, the teachers should identify the situations which link to students' everyday life or to other fields like finance, bank, medicine, sports, arts, and so on. The teachers can also facilitate their students' activities by giving them simple modelling situations at the beginning stage and more complex modelling tasks at the next stage.

The Modelling Process

There are many variations of the modelling processes; however, most of them are basically similar. Many researchers considered modelling as a multi-step process (Blum & Ferri, 2009; Galbraith, Stillman & Brown, 2010; Swetz & Hartzler, 1991). The process started with a real life situation or a problem that stems from other fields such as biology or physics by posing/asking a question or situation. The process continues with formulating conjectures and developing a model in mathematical terms and simplifications are made if necessary. In other words, the students need to calculate the measurement of given objects to identify the relationships among quantities and establish a function, an equation or system of equations. We call this phase of process mathematisation. Then the mathematical problem can be solved (solving an equation, graphing data, etc.). We call this step working with the model. Finally the results of the original problem must to be interpreted, validated, disseminated, and revised in a real context.

In Vietnamese mathematics classrooms, we applied the following seven-stage modelling process: (1) real-world problem; (2) make assumptions; (3) formulate mathematical problem; (4) solve the mathematical problem; (5) interpret the solution; (6) verify the model; (7) report, explain, predict. During the modelling process, the students must transfer among these steps such as understanding the realistic task, simplifying the task, mathematising, solving mathematical problems, interpreting results, and revising the model (Blum & Ferri, 2009; Kang &Noh, 2012). However, during the experimental teaching period, we also realized that different classrooms may have implemented the modelling process in a very different way.

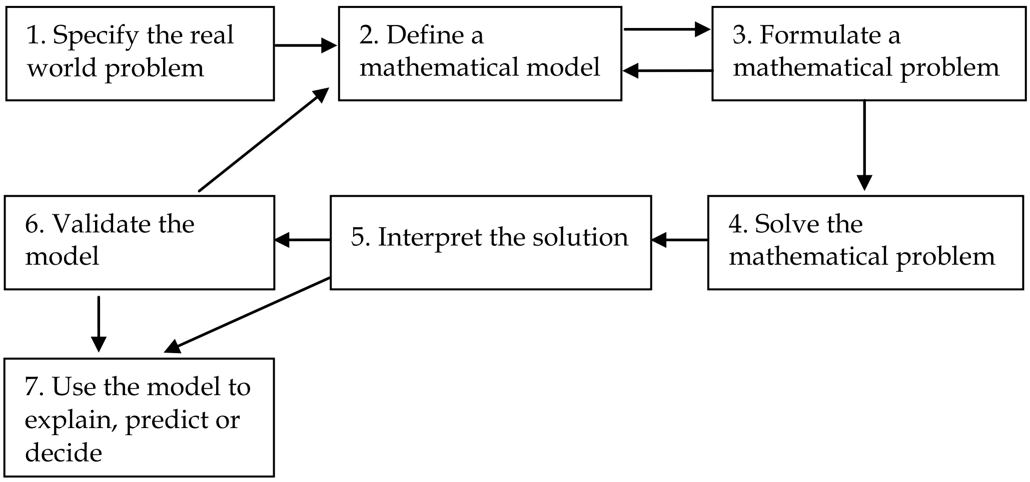

Figure 1: Main stages in modelling (adapted from Mason, 1988)

In figure 1, the teachers started the modelling process with a real world situation. Then the situation is re-structured and simplified in order to build a mathematical model. Then the students used mathematical languages to convert the model to a mathematical problem. Mathematical tools and methods are applied to attempt the problem. The result is reflected with the initial problem. The appropriate results are tested and verified. Finally, the model can be improved so that it can represent the situation better. Therefore, we used modelling in the classroom with the purpose of applying mathematics to: (1) understand observed phenomena in real life (e.g., engineering, physics, physiology, ecology, chemistry, economics, sports, music); (2) examine related questions about the phenomena; (3) clarify the phenomena in real context; (4) test hypotheses; and (5) predict about the real world.

Example 1. *Teacher gave students a photograph showing the motion of water spouting out from Merlion (a Singapore landmark). Then the teacher asked the students to use GeoGebra software to determine the model that represents the trajectory of the water.*

In this example, firstly the students chose the origin of the Cartesian coordinate system such that it coincided with the starting point of the water. The students predicted the shape of the motion (quadratic function) and created new value of parameter *m* using the slider of the software. Secondly, they typed the equation of the quadratic function in the form $y = mx^2$ in the input field. Thirdly, they moved the point on the slider (the parameter m) until the graph of the quadratic function overlapping the trajectory of the water. Finally, the students wrote down the found quadratic equation. Through these activities, the students could see the moving path of the water is a parabola that has the equation $y = -0.1x^2$. As a result, they get more understanding about some kinds of motions such as water spouting out from a high location or the falling of a ball. Parabolas also represented some types of building such as an arch bridge, the motion of some planets around the sun, etc.

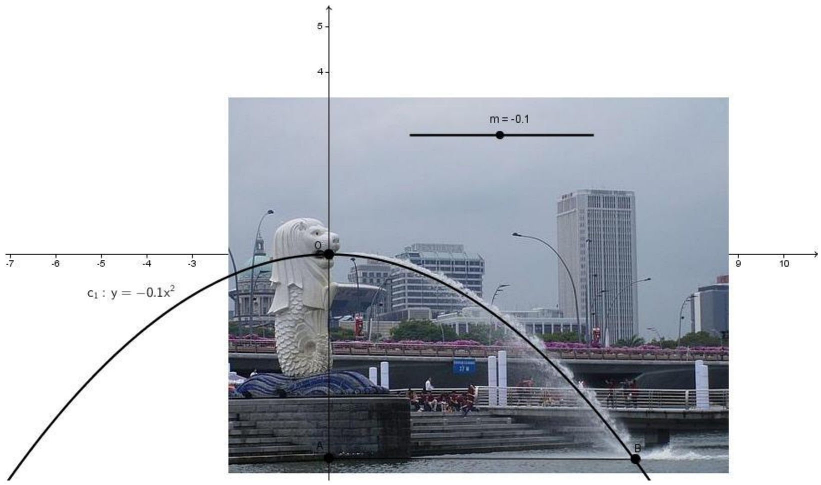

Figure 2: Modelling the trajectory of the water from Merlion using GeoGebra

Example 2. *The number of human population was calculated by the formula $S = Ae^{rN}$, where A is the population of the starting year, S is the population after N years, r is the annual population growth rate of the year. We know that in the year 2001, the population in Vietnam was S = 78.685.800 and the growth rate was r = 1.7% in that year. When will the population of Vietnam reach the number of 100 million people if the growth rate does not change?*

This modelling task was used to help the students get more understanding about the applications of exponential growth in real life. In lower grades, students were taught about linear and quadratic growth. The purpose of this example is to provide the students with an insight into the distinction among graphs of linear, quadratic and exponential functions that are representing growth rates. In this problem, the students could realize that at the beginning stage of time the graph's linear and exponential functions are nearly similar. In other words, there is no difference between linear and exponential models. Nevertheless, at the later stage of time (after 10 years), there is an enormous dissimilarity between the graph of linear and exponential function.

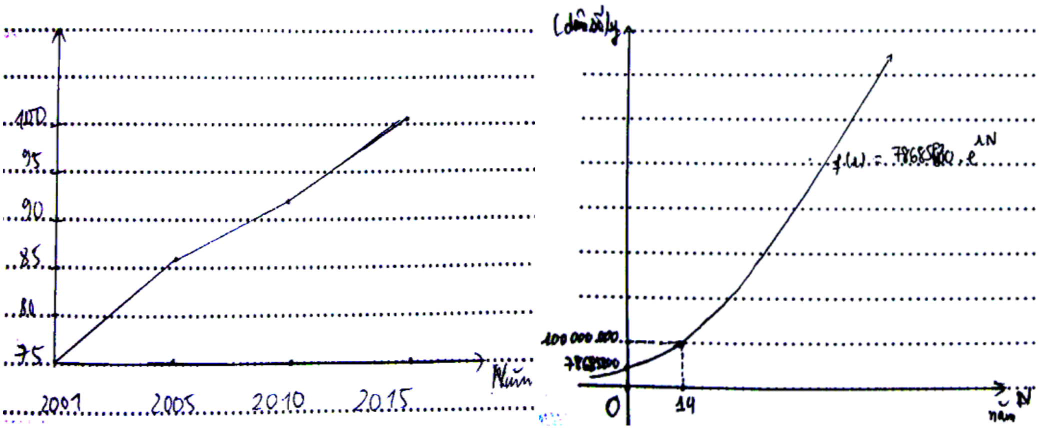

Figure 3: Modelling exponential growth of Vietnamese population

Stage 1 (*Real world problem*): The problem was stated very explicitly with the mathematical model $S = Ae^{rN}$. As a result, students did not need to collect data of population in some years in order to formulate a model. However, the students understood that this problem is very close to their real life.

Stage 2 (*Make assumptions*): Based upon the model, some students made conjectures about the growth of population by drawing a graph of the exponential function. They knew that the number of populations would increase very quickly and reach the number of 100 million in a short time.

Stage 3 (*Formulate mathematical problem*): Most of students could write the exponential equation with one variable N: $78685800.\ e^{0.017N} = 100000000.$ The problem now is to solve the equation to find the value of N.

Stage 4 (*Solve the mathematical problem*): By solving the equation, the students found that $N \approx 14$. The solution was presented as follows:

Theo bài ra ta có: (We have:)

$78.685.800 \, . \, e^{0,017.N} = 100.000.000$

$\Rightarrow e^{0,017.N} = \dfrac{100.000.000}{78.685.800} \approx 1,27$

$\Rightarrow 0,017 \, . \, N = \log_e(1,27)$

$\Rightarrow N \approx 14$

Vậy đến năm 2015 dân số nước ta là 100 triệu người.

(Therefore, the Vietnamese populations are 100 million in 2015)

Figure 4: The solution of the exponential equation illustrating population growth

Stage 5 (*Interpret the solution*): From the result $N = 14$, the students derived that the Vietnamese population will reach the number of 100 million in 2015. Most of students could get this answer but there were some students who did not comment more about the final result and compare the result with the real data about the population.

Stage 6 (*Verify the model*): There were only some students who showed a good connection between the solution and the real situation. They said that the initial model was not suitable: "*The population in Vietnam in 2015 is about 90 million people. This number was not equal to the result of the calculated model. This error stems from growth rate ... In fact, growth rate also depends upon variety of factors such as immigration and emigration rate, war, population policy, etc. However, I can realize that the population policies in Vietnam were implemented successfully in the past decade....*". It means that some students were able to verify the accuracy of the model in the real situation and realized that it is necessary to revise the model of exponential function.

Stage 7 (*Report, explain, predict*): Some students explained the difference between the solution and real life data because of the changing growth rate in every year. They predicted that this rate will be decreased in the next few years and then warned about some disadvantages of this falling trend to the state of national economics.

To sum up, modelling approaches provide students with a learning environment where they are invited to investigate, by means of mathematics, situations arising in other areas of knowledge. In particular, by designing mathematical models, the teachers can integrate the knowledge of mathematics in tackling important social issues such as population rate growth, environment protection, climate change, disease spread, etc.

Collecting the Data

In sum 180 students from different high schools participated in this research. Students were asked to work with some real-life modelling problems with restricted teacher support. Teachers gave only strategic interventions by using some kinds of requests like: *Make a sketch or diagram; Which data do you need? Which information do you need to collect? What does this model describe in the real life?* The teachers have designed supporting questions so that the students could solve the problem based upon seven steps of the modelling cycle. We divided each class into different groups and allowed them to discuss the problems with a system of focusing questions from the teachers. After school each group continued to solve other similar problems (normally project-based work) independently with the assistance of prospective teachers from a faculty of education. The groups also were allowed to choose an interest project topic to investigate by means of mathematics. It took several days, weeks or even months to complete are modelling projects. Data were collected through audio-recordings of group discussion and then were transcribed. Students' written protocols were also analyzed based on the seven-step modelling process. Finally, we conducted a semi-interview individually after each test aimed at getting more information about students' thinking and strategies during the process of solving the modelling problems. Teachers' took notes which were also used to record the students' difficulties during modelling process.

RESULTS

All 180 students participated in the all tests which include modelling problems and the results have shown that only 34% out of all of the students created correct mathematical models. In particular, 49% of the participants created incorrect models or solved the problem wrongly. The rest of them did not deal with it at all or were not able to create any mathematical model. They have met difficulties in transiting from real world problems into mathematics problems and finding a suitable model for the situation. We also realized that during every phase in the modelling process, the students had potential cognitive barriers. For example, in step 1, many students get stuck in interpreting the real life problem, translating into mathematical problem and building a suitable model. By interviewing, we realized that the students' difficulties stem from their lack of life experiences, especially students who lived in rural and mountainous areas. In step 2, the students were afraid of making assumptions by themselves; consequently they had difficulties in simplifying, structuring, and mathematising the problem. In particular, most of the students did not present any comments about validating the created model in real life. Through the interview, 61% of students said that they have learned brilliant strategies through the activities in which mathematics is currently being applied outside the classroom although 22% of them believed that the activities were boring and 6% said that mathematical modelling is very time-consuming because they have not encountered such a problem before. Until students are accustomed to this type of problem, they would not take the time to complete all seven steps in the modelling process. By analyzing students' written protocols, we have concluded that the modelling process is not linear and also not cyclic because the students can jump between the different stages in an unsystematic manner. In other words, it can be said that students' individual modelling tracks depended upon their individual preferences or problem solving strategies.

Through the interview, most of the teachers said that they met great difficulty in managing the classroom using a modelling teaching method, and the teachers have actually changed their role in the classroom from instructors to guiders and advisors. The teachers have also confirmed that a modelling activity should be based on an open, complex, realistic problem/situation. It should challenge the students' curiosity, encourage a deeper understanding of important mathematical ideas, and enhance individual thinking as well as group discussion. As a result, this approach develops the ability of inductive and deductive students' thinking and competencies like problems solving, formulating and testing of conjectures, revealing of causal relations and connections between related features. Teachers' skill of applying information and communication technologies in teaching mathematics (e.g., simulation, graphing, data analyses, etc.) was also a technical barrier in representing the modelling process. Finally, the teachers argued that they did not often use this method in teaching mathematics because they did not have any kind of book guide about this issue at all and they also were not able to realize modelling problems from real life situations.

DISCUSSIONS

In general, we have revealed that some barriers in applying this modelling method in the classroom linked to teachers' teaching styles, beliefs and teaching skills. Most teachers in Vietnam have little or no experience in mathematical modelling. They met difficulties in selecting and designing tasks that are open-ended, realistic and competency-based. They did not use this approach often because the modelling activity normally has time constraints in comparison to a traditional approach. It was also not easy to show students what to do and then guide them through practice. In particular, modelling requires the teachers to prepare a careful lesson plan and design a system of questions aimed to evaluating the students' model. All of the teachers agreed that this modelling approach would provide an opportunity for students to understand about the developing process of the mathematical concepts. By working with modelling tasks, the students could develop their mathematical skills, deepen their understanding of mathematical concepts and make a connection between mathematics and other areas, especially "very near to reality" situations that allowed interdisciplinary insights. The main difficulty with implementation of modelling in the curricula is that most of teachers are lacking experience of modelling both at the secondary school and at the teacher training university.

CONCLUSIONS

In conclusion, this modelling approach would provide students with a potential opportunity to connect mathematics knowledge in the classroom to their real life, school and society. The teachers should allow the students to analyze realistic situations, formulate and test the conjectures, choose and use appropriate mathematics tools and methods, build and interpret the mathematical model, reflect to real life and then to adjust their thinking. As a result, the students would utilize the created model to interpret the real world phenomena, make conjecture, produce arguments, and forecast about situations in the future. Therefore, we could consider mathematical modelling in the classroom as an active learning method and the students could learn mathematics in a meaningful way if this model was applied universally.

The following conclusions can be drawn from empirical findings that small group work on modelling problems may provide opportunities to introduce mathematical modelling and control students' activities in the classroom. However, the teachers must offer their students suggestions and private supports during the modelling process by giving the students proper guidance and scaffolding questions. In particular, the teachers need to design lessons that use both skill standards and modelling process practices. We also found that it takes a long time from empirical research to application in the classroom. Hence, the role of the modelling approach and its feasibility is an on-going discussion in the mathematics classroom. The results of this research also would make a contribution to modernizing the mathematics curricula and textbooks in Vietnam in which mathematical modelling as well as problem solving will be considered as students' core competencies.

REFERENCES

Ang, K.C. (2006). *Differential equations: Models and methods*. McGraw-Hill, Singapore.

Berinderjeet Kaur, Jaguthsing Dindyal (2010). *Mathematical applications and modelling*. World Scientific Publishing.

Biembengut, M. S., & Hein, N. (2010). Mathematical Modelling: Implications for Teaching. In R. Lesh, P. L. Galbraith, C. R. Haines & A. Hurford (Eds.), *Modelling Students' Mathematical Modelling Competencies* (pp. 481-490): Springer.

Blum, W. & Ferri, R.B. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, 1(1), 45-58.

Brown, S., & Walter, M. (2005). *The art of problem posing*. Mahwah, NJ: Lawrence Erlbaum Associates.

Dan, Q., Xie, J. (2011). Mathematical modelling skills and creative thinking levels: An experimental study. In G. Kaiser et al. (eds.), *Trends in Teaching and Learning of Mathematical Modelling*, International Perspectives in the Teaching and Learning of Mathematical Modelling. Spinger.

English, L. D. (2003). Reconciling theory, research, and practice: A models and modelling perspective. *Educational Studies in Mathematics*, 54, 2 & 3, 225-248.

Frejd, P., & Bergsten, C. (2016). Mathematical modelling as a professional task. *Educational Studies in Mathematics*, 91(01), 11-35.

Galbraith, P. L., Stillman, G., & Brown, J. (2010). Turning ideas into modelling problems. In R. Lesh, P. L. Galbraith, C. R. Haines & A. Hurford (Eds.), *Modelling Students' Mathematical Modelling Competencies* (pp. 133-144): Springer.

Gravemeijer, K., & Doorman, M. (1999). Context problems in realistic mathematics education: A calculus course as an example. *Educational Studies in Mathematics*, 39, 111-129.

Kang, O. K., Noh, J. (2012). Teaching mathematics modelling in school mathematics. *Proceedings of the 12th International Congress on Mathematical Education*, 145-157.

Kaiser, G. (2014). *Mathematical modelling and applications in education*. Springer.

Kaiser, G., & Stillman, G. (2015). *International perspectives on the teaching and learning of mathematical modelling*. Springer.

Lesh, R., & Lehrer, R. (2003). Models and modelling perspectives on the development of students and teachers. *Mathematical Thinking and Learning*, 2 & 3, 109-130.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modelling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 763-803). Charlotte, NC: Image Age Publishing.

Lingefjärd, T. (2006). Faces of mathematical modelling. *The International Journal on Mathematics Education*, 38(2), 96-112.

Maab, K. (2006). What are modelling competencies? *The International Journal on Mathematics Education*, 38(2), 113-142.

Maaß, K. (2004): *Mathematisches Modellieren im Unterricht. Ergebnisse einer empirischen Studie*. Hildesheim: Franzbecker.

Muller, E., & Burkhardt, H. (2007). Applications and modelling for mathematics —Overview. In W. Blum, P.L. Galbraith, H-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education*, The 13th ICMI study (pp. 267–274). New York: Springer.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Reston, VA : NCTM.

Niss, M., Blum, W. & Galbraith, P. (2007). Introduction. In W. Blum, P. Galbraith, H. Henn and M. Niss (Eds.), *Modelling and applications in mathematics education*, the 14th ICMI study (pp. 3-32). New York: Springer.

Nguyen, D. N., Tran, T. (2013). Recommendations for mathematics curriculum development in Vietnam. *Proceedings of the 6th International Conference on Educational Reform*, pp. 26-32.

Swetz, F. and Hartzler, J. S. (1991). *Mathematical modelling in the secondary school curriculum*. The National Council of Teachers of Mathematics: Reston, Virginia.

Zawojewski, J. S, Lesh, R., & English, L. D. (2003). A models and modelling perspective on the role of small group learning. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: A models and modelling perspective on teaching, learning, and problem solving in mathematics education* (pp. 337-358). Mahwah, New Jersey: Lawrence Erlbaum.

Zhu, K., Li, W., and Fu, X. (2013). Modeling population growth in online social networks. *Complex Adaptive Systems Modeling*, pp. 01-14.

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