Assessing the Influence of Augmented Reality in Mathematics Education: A Systematic Literature Review

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Abstract. One of the promising technologies to support the application of mathematics learning is augmented reality (AR). It is considered an important pedagogical tool that allows an increasing understanding of challenging ideas at most levels of education. This article presents the approach and concept of a systematic literature review (SLR) for reviewing the effects of AR in mathematics education. Filtering relevant material on AR and mathematics education from two databases (Scopus and Eric) to answer research questions is part of the review study. In the investigation, a total of 23 publications from 2018 to 2022 were systematically selected based on the PRISMA protocol. A review of the literature shows that interest in AR research has grown over time and is evenly distributed across different countries. The use of AR in mathematics education has been adopted and used as a supporting medium for interactive learning at various levels, from elementary school to college, that appears on the topics of geometry, algebra, basic mathematics, statistics and probability, and other mathematical topics. The effectiveness of AR, which is widely developed by researchers, is its ability to overcome existing problems, such as learning barriers, mathematical anxiety, and other cognitive problems. This review has filled and amplified the literature on AR on the effectiveness of AR in school mathematics learning. We recommend that in the future research on AR should focus on exploring the broad uses and long-term impacts of AR development and implementation on mathematics learning.

Keywords: augmented reality; learning media; learning technology; literature review; mathematics education
1. Introduction

The diversity of theoretical ideas regarding principles, methods, and topics in the mathematics education research community has its own uniqueness. Each idea has its own focus in mathematics education, prioritizing certain aspects (Font et al., 2011). Most theories in this field emphasize the complexity of the mathematical objects taught and learned (Mattis, 2015). However, in our view, the complexity of mathematical objects and the learning process is the key to answering the question of why there are many theoretical approaches in mathematics education. Alternative theories can help address the complex didactic problems in mathematics, as each theory can cover different aspects. However, using several theories with different assumptions and terms to approach the same problem can lead to conflicting results and hinder progress in mathematics education. In this case, the challenge lies in how to combine and integrate those theories into a framework that includes appropriate and adequate tools for the desired work (Moll et al., 2016; Prediger et al., 2008).

Among the many tools available for mathematics learning today, augmented reality (AR) has attracted many researchers. It is integrated with various learning theories and used as an alternative to overcome the complexity of mathematical material. Kaufmann et al. (2000) discussed the application of 3D constructs in mathematics and geometry at the high school and university levels. Dinayusadewi and Agustika (2020) applied AR to geometry materials for elementary school students, and Velázquez and Méndez (2021b) discussed the use of AR in algebra.

AR has experienced rapid growth as it is often adopted as an interactive technology option in various learning and education contexts (Nurbekova & Baigusheva, 2020). AR makes the teaching and learning process more flexible and simplifies complex knowledge (Hamzah et al., 2021). Aside from being widely used in education at all levels (Akçayir et al., 2016; Ponna & Piller, 2019; Thees et al., 2020; Weng et al., 2019; Wong et al., 2021), AR has been studied in a number of academic fields outside of mathematics, including physics (Thees et al., 2020), biology (Weng et al., 2019), and chemistry (Wong et al., 2021). One of the main factors contributing to its widespread benefits is the ability of AR technology to operate on various types of devices, such as personal computers, tablets, smartphones, and notebooks.

AR is a program that integrates virtual objects with the real world, as well as a tool that is interactive in real time (Azuma, 1997). In other words, AR is a tool used to add information and a view of the real world through virtual objects. In general, AR is used to connect visual objects and real environments to clarify and simplify the display of complex materials (Dunleavy et al., 2009). Since its first introduction in the 1990s, “mixed reality” — a term to refer to a combination of visual objects and the real world — has continued to receive considerable attention and study as a new training tool and teaching method (Caudell & Mizell, 1992). Although AR studies are gaining popularity among academics and researchers in the field of mathematics education, there is still limited knowledge regarding the usefulness of AR in the mathematical pedagogical.

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The aim of this paper is to investigate, evaluate, and categorize the literature that has been written about teaching mathematics through the use of AR. This comprehensive review examines the usefulness of AR in the field of mathematical pedagogy. Ibáñez and Delgado-Kloos (2018), Ajit et al. (2021), and Velázquez and Méndez (2021b) reviewed AR for STEM (science, technology, engineering, and mathematics), while Ahmad and Junaini (2020) and Jabar et al. (2022) studied AR in mathematics learning SLR with basic questions.

Table 1 illustrates the frameworks of some of the previous AR researchers. Based on the existing works in Table 1, it appears that a comprehensive and thorough analysis of the particular theme has not yet been presented.

Based on Table 1, the similarities observed between this study and previous research lie in the chosen protocol, specifically our use of the PRISMA protocol. Another similarity is related to the selection of the year and keywords. However, the most prominent difference is related to the research questions posed. We
argue that the research questions posed are the most critical element in an SLR. Our questions are focused on the development and utilization of AR in mathematics education, while other researchers focus on more general topics. Although AR technology has been around for a while, its potential in the field of education is currently being developed.

Unlike other computing technologies, AR provides an experience of limitless interaction between the virtual and real world. This serves as a metaphor for the real interface and also acts as a means of transition between the virtual and real world. SLR becomes important as a first step to explore how the characteristics of AR can be developed and effectively applied in formal educational environments, because SLR is a scientific technique for collecting all accessible information according to established criteria to address a specific research problem (Gough et al., 2017). In addition, SLR is also a systematic and appropriate method of classifying, selecting and critically analyzing various studies or research documents (Tikito & Souissi, 2019). Compared to traditional literature reviews, SLRs enhance review validity, reliability, replicability, and consistency (Xiao & Watson, 2019). An author’s claim of accuracy can be clarified by a methodical review, allowing gaps and directions for further research to be identified.

2. Literature Review

Modern technology is becoming increasingly important in modern culture as it helps to simplify daily tasks and provide quick access to information through various means. The number of jobs dependent on technology has also increased in recent years, making it important for children and teenagers to learn about technology from a young age. Currently, technology enables individuals to interact with it through simulated learning experiences. AR is a modern technology that enhances real-life experiences by incorporating virtual elements. Although AR is still considered a relatively new technology in the field of education, its benefits in the teaching and learning process are significant. Therefore, a research-based guideline is needed to design AR tools that are appropriate for school-based learning (Ozcakir & Cakiroglu, 2021). AR-enhanced creativity and motivation among students, realistic visualizations, improved 3D object visualization, rapid generation and manipulation of models, and ease of rotation.

Basically, AR can be defined as the process of adding new information through computer devices such as computers, tablets, or smartphones. When these devices detect certain patterns, positions, and images, they will display additional information that is added to the existing information in that reality. Azuma, an expert in studying AR technology, views AR as a combination of real elements and interactive virtual elements recorded in three dimensions and in real-time (Azuma, 1997). Technology-assisted learning and teaching through AR has several advantages. AR technology enables human-machine interaction to become more natural and provides a reliable real-world reference framework for users to perform specific actions (Velázquez & Méndez, 2021a). This process can be achieved by superimposing virtual objects onto the real environment. Students
can experiment with the ability to combine their actual environment with the created virtual environment.

Referring to its principles and technologies, there are many types of AR systems and applications. Different research publications classify categories in different ways, so there is no consistent classification for this group. The six different types of AR discussed by Edwards-Stewart et al. (2016) fall under two main categories: triggered and view-based augmentation. Marker-based AR, dynamic augmentation, location-based AR and complex augmentation fall into the category of triggered AR technologies while indirect and non-specific digital augmentations are included in view-based augmentation; and Rabbi and Ullah (2013) distinguished two groups of methods: marker-based and marker-less (location-based). These two primary types of AR serve various specific goals and were developed using various methodologies.

3. Review Methodology
The purpose of this study is to collect, assess, and synthesize empirical data on AR and its effects on mathematics learning through SLRs. SLR provides several potential benefits both to support further research efforts from the findings that have been presented by previous works (Kitchenham et al., 2009). This review procedure refers to what is recommended by Kitchenham et al. (2009), which we modified it to fit our framework. The modifications are related to the stages carried out, relating to the activities in each stage referring to what Kitchenham et al. (2009) explained. In general, the stages of implementing this SLR consist of three major parts: planning, development and results. The details of each stage are shown in Figure 1.

Figure 1: SLR phase diagram

During the planning stage, activities include identifying the review needs, determining research questions (RQ), selecting databases, searching for keywords, and determining inclusion and exclusion criteria. During the development stage, activities include conducting a primary study search without screening, screening studies based on inclusion and exclusion criteria, and extracting and synthesizing the data. During the results stage, activities include a quantitative summary of the findings, discussion, and conclusion. The 23 studies
from the Scopus and ERIC databases measuring the impact of AR on mathematics learning were selected for analysis.

3.1 Planning
In the initial planning phase, a review procedure is developed as a guide for reviewing and determining the main outcomes, methods and objectives of the SLR review. In this phase, keywords, inclusion criteria, exclusion criteria, and research questions (RQ) are identified. By using predetermined keywords, an article search was performed on the Scopus and ERIC databases. As a result, 568 journal articles (467 articles from Scopus and 101 from ERIC) were found based on title-abstract-keywords search: “augmented reality” OR “augmenting reality” OR “augmented reality learning” AND “mathematics” OR "mathematics education" OR "learning mathematics" OR "teaching mathematics”. We set inclusion-exclusion criteria (Table 2) to simplify the process of selecting appropriate literature.

### Table 2: Criteria of inclusion and exclusion

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Inclusion</th>
<th>Exclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Article title and content</td>
<td>An appropriate title that complied with the study’s requirements</td>
<td>Did not match the requirements of the study and had an irrelevant title</td>
</tr>
<tr>
<td>Year of publication</td>
<td>Publications from 2018 to 2022</td>
<td>Publications outside the range specified</td>
</tr>
<tr>
<td>Type of publication</td>
<td>Solely for journal articles</td>
<td>Reviews, editorials, and non-empirical studies</td>
</tr>
<tr>
<td>Language</td>
<td>English</td>
<td>Others</td>
</tr>
<tr>
<td>Field of article study</td>
<td>Mathematics education</td>
<td>Others than mathematics education</td>
</tr>
<tr>
<td>Accessibility</td>
<td>Full-text articles or open access</td>
<td>Preview articles and required a payment</td>
</tr>
</tbody>
</table>

RQ is the beginning and basis of SLR. RQ is used to guide the process of searching and extracting literature. Data analysis and synthesis, as a result of SLRs, is the answer to the RQ we specify up front. RQ formulations are presented in Table 3.

### Table 3: Research questions

<table>
<thead>
<tr>
<th>ID</th>
<th>RQ</th>
<th>Motivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQ1</td>
<td>How is the development of AR in mathematics learning based on the distribution of years and their demographics?</td>
<td>Knowing the year and demographics will provide an overview of the development of AR studies that have been carried out and predict what is still necessary and will be investigated next.</td>
</tr>
<tr>
<td>RQ2</td>
<td>Who are the target AR users in mathematics learning?</td>
<td>The population in selected studies can provide an overview of the most appropriate use of AR in terms of cognitive development levels.</td>
</tr>
<tr>
<td>RQ3</td>
<td>Who is an AR developer in mathematics learning?</td>
<td>This study will provide an overview of how far the role of interested parties is for the advancement of AR development and implementation in formal mathematics.</td>
</tr>
<tr>
<td>RQ4</td>
<td>How is the implementation of AR</td>
<td>The pedagogical aspect in the implementation of AR is intended to see how far AR becomes a tool that can lead</td>
</tr>
<tr>
<td>ID</td>
<td>RQ</td>
<td>Motivation</td>
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<td>----</td>
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</tr>
<tr>
<td></td>
<td>What problems can be solved using AR technology in mathematics learning?</td>
<td>This section offers on the side where AR can contribute more in solving mathematics learning problems</td>
</tr>
<tr>
<td>RQ5</td>
<td>What topics are featured in AR?</td>
<td>The extent to which AR can facilitate mathematical topics will be seen in this section</td>
</tr>
</tbody>
</table>

3.2 Development
The development stage is the stage that contains the implementation of the SLR, where we refer to the standard PRISMA. PRISMA creates a uniform, peer-reviewed technique that makes use of checklists of best practices to help ensure the quality and reproducibility of the revision process (Conde et al., 2020). Identification, screening, eligibility, and inclusion are the foundational elements of PRISMA.

3.3 Result
The final stage involves a methodical analysis and discussion of the reported results, which leads to the conclusion of the SLR. Trends, study deficiencies, and

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suggestions for further investigation are also mentioned. In order to properly assess the importance of the phases illustrated in Figure 1, and to underscore the methodological limitations inherent in conducting an SLR, it is necessary to conduct a thorough analysis.

4. Review Result and Discussion
The papers selected for this SLR were obtained from the Scopus and ERIC databases, as mentioned in the methodology section. These databases were chosen because they comply with protocol requirements and have a filtering feature that automates specific parameters set. As shown in Figure 2, a search using the specified keywords yielded 586 articles from both databases. Due to the application of inclusion criteria, 485 were excluded; there were 11 duplicate articles, which reduced the number of articles to 72. After careful review of “title, keywords, abstract and content”, 49 articles were excluded for not having a focus of study in “mathematics education”. Finally, a total of 23 scientific articles was analyzed.

All articles (n=23) were analyzed to gather the information we needed to answer our research topic, then the discussion was categorized into seven categories according to the research question. The following subsection provides answers to the research questions.

4.1 Distribution of research study by publication year and country
The first RQ relates to the year of publication and the country in which the research was conducted. Overall, there are a total of 23 related articles from the Scopus and ERIC databases published between 2018 and 2022. Figure 3 shows the distribution of studies analyzed by year of publication.

![Figure 3: Distribution of research studies by publication year](image)

It is interesting to note that interest in AR research has evolved over time. Overall, research related to AR increased in 2019 (n=5) and 2020 (n=6), resulting in the publication of a total of 11 papers. The study of the application of AR in the field of mathematics education is seen as a new direction from various studies that have been carried out previously. The number of publications published in 2021 increased rapidly to reach eight articles. However, only four articles were released in 2022. This growth suggests that researchers are increasingly interested in using AR as a medium for mathematical learning (Cahyono et al., 2020). According to
the Horizon report, AR is a new and rapidly advancing educational technology (Brown et al., 2020), and it is possible that, in the coming years, research supporting this evolving technology will increase.

AR is predicted to become increasingly popular and develop among the public, including in the educational environment (Martín-Gutiérrez et al., 2017). Several non-educational applications that use AR have become popular trends, such as the game Pokémon GO, which has attracted the interest of both adults and children, demonstrating the potential of AR usage. In the context of mathematics learning, there has been an increase in AR usage in 2021 that may be related to the impact of the COVID-19 pandemic (Eldokhny & Drwish, 2021). The year of 2020 was the beginning of massive transmigration in all sectors of life — including education — due to the impact of the pandemic (Hendriyanto et al., 2021), thus forcing everyone to learn adaptively, of which, one way is through digital technology instruments. There has been a tremendous acceleration in the use of digital technology in the world of education during the pandemic.

With regard to the distribution of AR studies by country, based on Figure 4, it can be observed that there are publications from selected countries included in this study. According to the established criteria, Indonesia has the highest number of publications (n=8), followed by Spain (n=3), Malaysia (n=2), and Turkey (n=2). However, there are several other countries that only have one publication each, such as Beijing, China, Ecuador, Germany, Jordan, Mexico, Saudi Arabia, and Ukraine.

The researchers from each of these countries are:

1) Researchers from Indonesia: Fatimah et al. (2019); Amir et al. (2020a, 2020b); Cahyono et al. (2020); Sudirman et al. (2020); Wangid et al. (2020); Mailizar and Johar (2021); Wiliyanto et al. (2022).
2) From Spain: Flores-Bascuñana et al. (2020); Cabero-Almenara et al. (2021); Velázquez & Méndez (2021a).
3) From Malaysia: Ahmad and Junaini (2022); Hanid et al. (2022).
4) From Turkey: Ibili and Billinghurst (2019); Ozcakir and Cakiroglu (2021).
5) From Beijing: Cai et al. (2019).
6) From China: Li et al. (2022).
8) From Germany: Schutera et al. (2021).
10) From Mexico: Moreno et al. (2021).
12) From Ukraine: Vakaliuk et al. (2020).

The reason why developing countries apply ICT in education is to support various goals such as education reform, social progress, and economic development (Kozma & Vota, 2014). Many countries have used ICT as a means to train students’ skills and knowledge, even as if it had become an obligation in developed countries to involve ICT in learning. ICT integration in the education sector has improved dramatically worldwide over the past 40 years (Chen et al., 2020). ICT can complement, enrich, and transform education for the better (Garzón & Acevedo, 2019). Countries that have initiated ICT integration programs in the education system are Portugal — a program of one-to-one laptop schools (Lucas, 2018) in the Magellan project (Piper et al., 2017); South Korea — smart education program (Leem & Sung, 2019); Australia — digital education revolution program (Brown, 2021); Turkey — FATIH program (İra et al., 2021); one laptop per child program in Peru and Uruguay (Hennessy et al., 2021); and the O-OLS program in Latin America (Capota & Severin, 2011). Thus, the discovery of a AR-based learning approach demands further research in different countries around the world (Cahyono et al., 2020).

4.2 Educational level study sample
Research conducted by Neofotistos and Karavakou (2018), revealed that most of the students (junior and senior high school levels) master ICT well. This happens because ICT has been known by students since elementary school. The components of ICT are very important for education, because their use can support the smooth teaching and learning process and ICT can present opportunities for teacher-students to innovate on content, methods, and pedagogy (Zhang et al., 2016). AR in mathematics education has been implemented and applied as a medium to support interactive learning at various levels ranging from elementary school to college. The distribution of AR studies according to education level can be seen in Figure 5.
The data in Figure 5 demonstrate that junior high school level has the highest quantity of AR-based teaching media (n=10), followed by elementary school (n=5), college (n=4), senior high school (n=3), and finally specific learning disabilities (n=1). The transfer of thinking that is still contextual at the elementary school level to the senior high school level and which uses an abstract thinking perspective in mathematics requires a bridge that mediates; this makes some researchers interested in developing this AR-based media at that level. Next, elementary school students have a level of contextual thinking, so that, in learning, the teacher must always involve contextual problems (Phonapichat et al., 2014). This is inversely proportional to high school students who are required to have abstract thinking in mathematics (Reys et al., 2007).

Similar findings conclude that K-12 students make up the majority of the sample in AR-related articles (Akçayır & Akçayır, 2017; Ibáñez & Delgado-Kloos, 2018). K-12 children are the most desirable sample population, probably because they are in a period of stable functioning according to Piaget’s theory of human development (Kohler, 2014). At this stage, children can easily learn about concrete concepts through the process of reasoning and classification that involves multiple senses (Ghazi et al., 2016). As a result, learning tools like AR can help make abstract concepts more concrete and accessible during this learning phase. Despite mathematics being an intrinsic part of our daily lives, understanding abstract concepts remains a challenging topic in mathematics education at all academic levels (Lozada-Yáñez et al., 2019).

4.3 AR developer

AR technology is a new breakthrough in learning media where the learning process in the world of education so far is still mostly conventional. This can lead to a level of saturation and lack of motivation for students to learn. Technology will never replace the role of teachers, but teachers who do not take advantage of technology will soon be replaced. With advances in science and technology, teachers are expected to carry out their duties adaptively, innovatively, creatively and critically in the learning process. The teacher has full control over the implementation of mathematics education.
As facilitators of learning activities, teachers have the freedom to design learning activities that can be applied in their respective classes, both in a physical and virtual sense (Cuendet et al., 2013). The teacher’s role in developing learning activities must be able to engineer learning experiences that are interesting, varied, repetitive, and enhanced for students. The integration of ICT in mathematics learning has epistemic potential that allows students to engage in the instrumental genesis of mathematical concepts (Moreno & Llinares, 2018), and allows teachers to develop their abilities to achieve specific learning objectives (Stein et al., 2020), redesign learning and deliver new mathematics assignments to students (Yeh et al., 2021).

However, the fact that there is currently a competency gap between users and developers also needs to be taken in consideration. Recognizing that self-setting aside as a developer requires knowledge and skills, several strategies have been applied in programs for pre-service teachers, as for instance, in the curriculum of educational lectures and the teacher professionalism program, which is also known as the Program Profesi Guru (PPG) in Indonesia. AR developers have published the results, including those also aimed at learning mathematics. The teachers, however, are more inclined to the user.

With open access and opportunity to technology, in-service teachers also have a wide range of opportunities to use, or even develop their own ideas and products and disseminate them. Although it is not the main task of teachers to play the role of developers, they must still be given a platform for their ideas and competencies so that the integration of technology in education will be a transformation that goes is concurrent with its development. It is also relevant to how the research group has discovered the many benefits of developing and applying AR technologies in learning. The findings suggest that AR developers are currently dominated by researchers (n=21) (see Figure 6).

Figure 6: Distribution of AR developer

In the era of rapidly advancing communication and information technology, education must keep up with these developments by adopting teaching methods that are appropriate and compatible with current technology (Elsayed & Al-
Najrani, 2021). Following the ongoing digital developments, teachers are expected to be able to prepare themselves at least adequately to use the available technology to be applied in learning. The greatest opportunity must be provided for the development, implementation, and collaboration strategies that may be implemented.

For more serious conditions, looking at how problems occur in conventional learning, the use of ICT in education can be one of the pragmatic but sustainable ‘urgent solutions’. The utilization of ICT, especially AR, can be integrated into learning designs that are tailored to the current and future learning needs of students. This can be observed from students’ attachment to devices such as tablets, smartphones, and other technologies, which has driven the use of digital technology in education. Therefore, it is important for teachers to receive training in AR technology development (Sáez-López et al., 2020). Although the new challenge with technology is not only the economic cost but also the need for teachers to get used to it, this can be overcome more effectively to implement this technology in the classroom (Fernández-Enríquez & Delgado-Martín, 2020). Currently, only GeoGebra AR can be utilized by teachers for AR-based learning.

The role of governments to provide facilities that support the achievement of ICT integration in learning holistically is urgently needed.

4.4 The role of AR in learning mathematics
The role of digital technology in education is not only limited to its use as a learning tool, but encompasses various complex dimensions. Educational technology can be described as the study of ethics and practices that facilitate learning and improve performance through the appropriate use of technological processes and resources, as defined by the Association for Educational Communications and Technology (AECT) (Januszewski & Molenda, 2008). However, this research shows that the role of augmented reality (AR) in learning is only as an assistive tool (n=21) to enhance effectiveness and student engagement in the learning process; two other studies (Ozcakir & Cakiroglu, 2021; Wangid et al., 2020) developed AR as teaching materials (n=2).

To achieve learning goals, student performance is a very important factor. The selection of appropriate technology designed for learning must be tailored to the conditions and needs of students in their respective schools or regions. AR technology has certain advantages, such as promoting motor learning and supporting the learning process through the integration of digital learning elements that help engage students and enhance their motivation to learn (Diaz et al., 2015). AR technology can be easily implemented in various learning media, such as smartphones and printed materials like books, making it more accessible and practical to use (Wiliyanto et al., 2022).

Dunleavy and Dede (2014) stated that AR technology is considered to have the potential to enhance learning based on two different and independent theoretical frameworks. The first is the constructivist learning theory developed by Bruner and Vygotsky. According to this theory, individuals create new knowledge based on their prior knowledge. These theories view AR as a technology with great potential to enhance students’ ability to construct knowledge. Second, according
to the situational learning theory, meaningful learning occurs in a certain environment, and the effectiveness of such a setting is influenced by the interaction between people, objects, locations, processes, and cultures (Dunleavy & Dede, 2014). The use of AR provides a new way for students to connect with course material, teachers, and other students, as well as with activities, locations, and cultures that may be useful for learning.

4.5 Recommendations for AR to solve the problems in mathematics learning

ICT integration has become an integral and inclusive part of the educational process (Nikolić et al., 2019). The cone of experience model initiated by Dale (Edgar, 1970) gives an idea that the more concrete the learning experience that is passed, the higher the students’ understanding of the information they obtain. The more abstract the learning experience experiences, the less understanding is gained. In this case, ICT — AR technology — can provide a concrete learning experience in mathematics learning. It should be noted that the studies that have been carried out prove that the effectiveness of the use of AR in mathematics education is able to overcome existing problems (see Figure 7).

Figure 7: Distribution of AR to solve the problems in mathematics learning

Regarding the didactic proposals outlined, it can be said that the use of AR technology is an educational innovation, which can make a positive contribution to improving the understanding of geometry concepts, developing spatial visualization (Ahmad, 2021; Amir et al., 2020b; Elsayed & Al-Najrani, 2021; Flores-Bascuñana et al., 2020; Ozcakir & Cakiroglu, 2021; Schutera et al., 2021; Vakaliuk et al., 2020; Velázquez & Méndez, 2021a), and increased student motivation (Fatimah et al., 2019; Lozada-Yáñez et al., 2019; Mailizar & Johar, 2021; Sudirman et al., 2020). Student motivation and their involvement in learning is key to achieving effective learning (Fernández-Enríquez & Delgado-Martín, 2020).

The use of AR technology is beneficial for students through increased achievement (Moreno et al., 2021) and learning performance (Cabero-Almenara et al., 2021; Wangid et al., 2020; Wiliyanto et al., 2022). AR helps with knowledge construction (Amir et al., 2020a; Cahyono et al., 2020; Cai et al., 2019; Ibili & Billinghurst, 2019), develops thinking skills (Li et al., 2022) and supports learners to better understand the topic being studied (Cabero-Almenara et al., 2021; Moreno et al., 2021).

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The way of thinking of students who have studied with AR tends to focus on developing their ability to translate embedded visual language in visual form (Amir et al., 2020b). Further results related to visuals are also obtained in solving one of the problems that are widely found, such as Velázquez and Méndez (2021a) and Ozcakir and Cakiroglu (2021), who found correlations of AR use and improved student spatial ability.

Students exposed to learning with GeoGebra AR — a learning package developed by researchers — in the experimental group, got better results in visualization and spatial rotation skills, compared to students in the control group. Therefore, Velázquez and Méndez (2021a) also recommend the use of AR as a support in the learning and teaching process to improve the performance of spatial abilities and, of course, also the academic performance of students.

In its application, AR accessed in the form of an application, which is standalone and made with an easy-to-use design, can facilitate students in the understanding of mathematics learning; as Ozcakir and Cakiroglu (2021) concluded, the application actively invites students who can interactively use the components in it directly, such as changing image types and parameters, and testing them simultaneously. AR builds spatial imaginations that can be harder to realize with two-dimensional teaching materials as usually used.

On the topic of spatial ability, an important finding shows that students contribute to their own development of spatial ability. According to Schutera et al. (2021), the support of AR can create positive perceptions and motivations in students during spatial-based learning activities, such as representation, visualization, rotation, reconstruction, and constructive space. Through these activities, students’ spatial ability can improve. This finding is also supported by Ahmad’s (2021) theory, that of the two-sided brain, in which there are two complementary methods in processing information. The second method shows how the brain works to find spatial relationships formed and is done in the right brain, both moving linearly, so they move step by step. When performing activities that require visual thinking, the brain also increases its activity in performing activities that require verbal thinking.

Discussing the use of tools in the learning process cannot be separated from the design used. In some cases, as in Flores-Bascuñana et al. (2020) and Vakaliuk et al. (2020), neither the characteristics nor designs described in the study are intended to be specific about the design and development of AR applications to encourage the development of spatial ability. However, they demonstrate how to use AR to provide learning opportunities that enhance students’ spatial abilities, while also helping teachers implement better classroom instruction.

In another perspective, another benefit of the use of AR when the goal has been determined in overcoming problems related to spatial ability, is that students will be able to use the tool to perform their spatial ability through spatial tasks and, with the use of their devices, it will be seen to be more supportive of students in enhancing the use of their devices more positively. Students will not only use their
devices to communicate, read books, or play games, but they will also become a support mechanism for students to learn mathematics (Vakaliuk et al., 2020). Elsayed and Al-Najrani (2021) supported that focusing on the implementation of AR in mathematics education can be a powerful tool to address students’ spatial ability issues, while simultaneously strengthening their spatial visualization skills.

Furthermore, the use of AR also demonstrates its potential in promoting student representational ability, described by Li et al. (2022) as an AR-based multi-representational learning environment (AR-based MRLE). AR-based MRLE promotes lower secondary school students’ representational ability in linear function. In the study, a successful experiment was to combine function material with its representation with AR that attempted to be exploited with a real-life dimension to representational learning of linear function. The three representations used are real-life, symbolic, and graphical.

Furthermore, in the context of mathematical modeling there are several ways that students use it to solve mathematical problems based on their work results. Mathematical concepts can be learned by students through mathematical modeling in various ways. Research by Amir et al. (2020a), in line with Sudirman et al. (2020) and Wangid et al. (2020), aimed to develop an AR system on mobile devices to improve students’ understanding of mathematical concepts. Concepts such as doing, drawing, having a picture, paying attention to properties, formalizing, observing, arranging, and discovering are involved in this topic.

The utilization of AR technology on smartphones can have a positive impact on learning outcomes, especially in financial mathematics. The use of AR technology can also enhance students’ perception of their environment and their interaction with it. AR technology alters the way in which students engage with the surrounding environment and offers a distinctive and interactive delivery of information, thus increasing their involvement in the process of learning. Although a three-dimensional model is required to represent AR, an alternative representation of the application can still motivate students, as reported by Vakaliuk et al. (2020). Furthermore, research has identified five computational skills, namely abstraction, generalization, decomposition, algorithms, and debugging, that can be used to solve geometry problems.

Computational thinking with the help of AR technology can be an effective approach to promote the use of technology in learning and trigger further research on technology and pedagogical approaches to solving problems in learning activities. Ibili and Billinghurst (2019) discussed the relationship between the use of AR teaching software and cognitive load. The study aimed to investigate the correlation between perceived usefulness, perceived ease of use, natural interaction, and intrinsic, extraneous, and germane cognitive load.

When discussing problem-solving and skill development in learning mathematics, student motivation is crucial, especially in the context of AR or technology use. Some articles discussed how student motivation is measured and
the results showed that student motivation developed in learning activities is more determined by the students themselves when using AR (Cahyono et al., 2020; Cai et al., 2019; Mailizar & Johar, 2021). Positive results in those cases indicate that the forms of internalized motivation (IM) and identified regulation (IR) are more dominant. Students feel that their learning activities are interesting and meaningful, and they are satisfied with the activities. They also learn how to apply mathematics in the real world with the help of mobile applications.

In a more serious context, mathematics education programs focus on activities that support students in building their own mathematical knowledge during their learning process. Cai et al. (2019) discussed how the utilization of AR apps in the classroom can facilitate mathematics learning for students who possess high levels of self-efficacy by adopting profound strategies. Research by Lozada-Yánez et al. (2019) described the use of MS-Kinect as its AR form. The result is that MS-Kinect can be used as an interactive device that provides various possibilities in its application to educational environments.

AR also helps in creating simplified representations of multidimensional objects used in educational content to facilitate students’ understanding. The idea of broad concepts and spaces has several problems in learning, such as examples in circles, ellipses, parabola, and hyperboles (Fatimah et al., 2019). Therefore, AR, with the integration of these topics that are adjusted to learning standards and media, will be generally attractive, easy to operate, facilitate understanding and, of course, will increase student motivation.

The use of AR in learning attitudes has a positive impact on learning motivation, as found by Sudirman et al. (2020). The use of local wisdom in AR can stimulate students’ curiosity in exploring geometric concepts, to pay attention to learning spirit, and encourage students to apply AR when learning independently. The four motivational factors that have been implemented are attention, relevance, self-confidence, and satisfaction. Researchers have integrated local knowledge into AR technology to enhance geometry teaching. They also analyzed how this affects students’ learning attitudes, motivation, and ability to understand geometric concepts.

Wangid et al. (2020) showed that mathematics anxiety can be a hindrance for students in achieving mathematics learning achievement. However, AR can help reduce students’ anxiety through its use in AR-assisted storybooks, which can have a positive and significant impact on students’ mathematics anxiety. Another advantage of using AR is that it helps students’ spatial abilities, which ultimately can reduce student anxiety. In understanding mathematics, students with learning disabilities require longer and repetitive time to understand concepts. Mathematics learning is also an abstract subject and requires constant repetition for students with specific learning disabilities (SLD). Therefore, AR is used as technology in mathematics learning in junior high schools to overcome SLD mathematical barriers by projecting images in three dimensions (Wiliyanto et al., 2022).
4.6 Mathematical topics used in implementation of AR

In relation to the mathematical topics used in the application of AR, five main areas of mathematics were identified: geometry (n=15), algebra (n=3), basic mathematics (n=2), statistics and probability (n=1), as well as other fields including mathematical economics (n=1) and mathematical engineering (n=1).

Table 4: Topic of implementations of AR in mathematics

<table>
<thead>
<tr>
<th>Mathematics Topics</th>
<th>n_i</th>
<th>Sample Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra</td>
<td>3</td>
<td>Li et al. (2022)</td>
</tr>
<tr>
<td>Basic mathematics</td>
<td>2</td>
<td>Lozada-Yáñez et al. (2019)</td>
</tr>
<tr>
<td>Geometry</td>
<td>15</td>
<td>Schutera et al. (2021)</td>
</tr>
<tr>
<td>Mathematical economics</td>
<td>1</td>
<td>Moreno et al. (2021)</td>
</tr>
<tr>
<td>Mathematical engineering</td>
<td>1</td>
<td>Cabero-Almenara et al. (2021)</td>
</tr>
<tr>
<td>Statistics and probability</td>
<td>1</td>
<td>Cai et al. (2019)</td>
</tr>
</tbody>
</table>

According to the findings of this study, Table 4 shows that the use of AR in mathematics education is more commonly applied to geometry, especially 3D geometry. This is due to AR’s advantages in visualization (Behringer et al., 1999) and the availability of 3D-based AR support software such as GeoGebra 3D; Unity; Assembler Edu, AR-Math, and Vuforia Augmented Reality SDK. Geometry itself is an important topic in mathematics (Ma et al., 2015) that studies shape, position, and spatial properties. While innovation in education does not always involve new technology, visualization issues in geometry cannot be addressed solely by using traditional manipulative teaching materials such as polyhedral made of paper and wood, or knot and end assemblies.

AR is frequently used to teach geometry because it makes abstract concepts tangible, enhances spatial visualization skills, and facilitates active learning. Geometry involves abstract concepts, such as points, lines, angles, and shapes, that can be difficult for students to grasp. AR provides a way to make these abstract concepts more tangible by allowing students to see and interact with them in a virtual environment. This can enhance their understanding of these concepts and help them visualize them more clearly. Spatial visualization skills are important for understanding and solving problems in geometry, and AR can help students develop these skills by providing them with opportunities to manipulate and explore geometric shapes and structures in three-dimensional space. AR also provides an immersive and interactive learning experience that encourages students to actively engage with the material, leading to greater retention, understanding, motivation, and engagement.

5. Conclusion

All research questions have been evaluated in this study. In terms of quantity, Indonesia appears to lead among the 23 selected articles, with eight studies obtained from the two databases that we used. The use of AR in mathematics education has been implemented and applied as a medium to support interactive learning at various levels ranging from elementary school to college. The results of the analysis revealed that junior high school students made up most of the sample in the review of selected articles, followed by students in elementary school. Piaget’s human development theory provides support for this notion, as
learners at this stage can easily grasp concrete concepts through cross-sensory classification and reasoning. The application of AR technology in mathematics education has been recognized as a “tool” that can be utilized in various topics, such as geometry, algebra, basic mathematics, statistics and probability, among others. AR, developed by researchers, has been proven to be effective in addressing various problems, including learning barriers, mathematical anxiety, and cognitive issues. AR technology is a useful and efficient tool that can be extensively applied in education, especially in mathematics education.

According to this study, if AR is to be used in a learning context, it is very important to have a clear and accurate conceptual characteristic of AR. The results of systematic studies strongly suggest that AR can be developed and implemented in pedagogical practice when knowing exactly the characteristics of AR that are suitable in mathematical learning.

This review has filled in and amplified the literature on AR on the effectiveness of AR in school mathematics learning. AR can be used to facilitate student engagement in learning but must still pay attention to alignment on implementation practices and materials that match AR technology. We recommend that, in the future, research on AR should focus on exploring the broad uses and long-term impacts of AR development and implementation on mathematics learning.

6. Limitations
Only indexed articles in Scopus and ERIC databases were used to review this investigation and even then it was limited to the last five years’ review. Future studies may also be able to use other databases, such as SCCI, ProQuest, and Springer. In addition, the study is restricted to studies published as articles. Future studies may focus on looking at a larger range of aspects, including conference papers, editorials, theses, and dissertations, as this may help researchers learn more about the benefits and disadvantages of using AR technology in mathematics teaching.

In contrast, there are a number of studies that do not address the uses and uses of augmented reality learning in mathematics education in detail. Therefore, the conclusions of this review are limited to some other studies with a clear justification.

7. References


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