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Mathematical Proficiency, Scientific Reasoning, Metacognitive Skills, and Performance of Learners in Physics: A Mathematical Model

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Abstract. This study aims to explore the mathematical proficiency, scientific reasoning, and metacognitive skills that contribute to the student's academic performance in Physics. Data were gathered quantitatively by administering the Basic Skills Diagnostic Test, Lawson's Classroom Test of Scientific Reasoning, and Metacognitive Awareness Inventory to Grade 10 students in the selected high schools of District II of Ifugao Province from January to June 2023. Using the Raosoft calculator, 369 out of 405 students were randomly selected as participants. Relationships of the variables were analysed using Structural Equation Modelling (SEM) where the direct and indirect effect was shown. Based on the model, mathematics proficiency, scientific reasoning (Formal Operational, Late and Early Transitional reasoners), and metacognitive skills (declarative, debugging, and evaluation) had a direct effect on academic performance in Physics. The scientific reasoning skills – late transitional directly affect the metacognitive skills – procedural, conditional, planning, and information. Further, an indirect effect of mathematics proficiency on scientific reasoning and metacognitive skills, scientific reasoning to mathematics proficiency and metacognitive skills, and metacognitive skills on scientific reasoning and mathematics proficiency, toward academic performance was observed. Conclusively, a strong foundation in mathematical proficiency with the effective use of scientific reasoning and metacognitive skills were found to be key determinants of success in Physics. This suggests that educational institutions craft Physics curricula considering the development of students in mathematical proficiency, scientific reasoning skills, and metacognitive skills, while teachers develop activities that refine the direct and indirect effects of these variables to target a better academic performance in Physics.

Keywords: Metacognition; Scientific Reasoning; Mathematics Proficiency; Physics; Modelling

1. Introduction

Physics is one of the fundamental sciences that help in understanding the universe and phenomena using mathematical context (Cleugh, 2018). Despite its importance, many students find Physics difficult, specifically in grasping the problem and utilising mathematical concepts (Sartika & Humairah, 2018; Reddy & Panacharoensawad, 2017; Yusrizal, 2016. Physics teachers fail to measure the student's ability in higher order thinking skills being exemplified in Physics (Saepuzaman et al., 2022. One of the major reasons for the students' perceived difficulty in grasping the concepts of Physics is the difficulty students have in connecting the principles of Physics to real-world phenomena (Nordin, 2019) along with problems with the students themselves, the curriculum, and subject-related factors (Wangchuk et al., 2023).

Currently, the Philippines faces a problem in science education as evidenced by the latest result of the Programme for International Student Assessment (PISA) in 2022, in which there was a significant absence of top-performing learners in Mathematics, with almost none reaching the highest performance levels (Levels 5 or 6), compared to the OECD average of 9%. In terms of science, the Philippines has one of the lowest scientific performance scores among participating countries and economies. The mean science score was 356, placing 78th out of 80. Both boys and girls did poorly in science; with boys placing 79th (349 score) and girls ranking 77th (363 score). Furthermore, the percentage of low performers in science, defined as those below competence Level 2, was significantly high, at 77.2%, placing fourth out of 80. Particularly alarming was the large percentage of low-performing boys in science, which reached 80.5% and ranked third out of 80 (Philippines Student Performance - PISA (2022), 2023). According to several studies conducted in the Philippines, the majority of students struggle in Physics for several reasons. Common problems include students' lack of motivation, poor mathematical abilities, and comprehension, as well as institutional challenges (Andres et al., 2015; Guido & Orleans, 2021).

Metacognitive skills and scientific reasoning skills predict students' success in Physics. Yanti et al. (2017) established that students with low metacognitive skills have difficulties with Physics problems from both easy and challenging questions, reasoning, solution making, and deducing outcomes, among others. However, positive metacognitive skills can boost students' attitudes toward the subject of Physics, specifically in quantum physics (Dökme & Ünlü, 2019). In terms of scientific reasoning, Fabby and Koenig (2014) found that learners with higher scientific reasoning perform well in Physics, specifically on algorithms and conceptually based problems. Also, Erlina et al. (2018) concluded that high reasoners outperform low reasoners; however, students' confidence can lead to poor scientific reasoning still resulting in low scores in Physics.

While there are studies on the association between characteristics such as mathematical skill, scientific reasoning, metacognitive ability, and academic achievement in Physics, research gaps still exist in that most studies examine how each component affects students' academic success in Physics. However, further study is needed to investigate the intersectionality of these characteristics to provide insights for Physics curriculum planners, teachers, and students to provide a deeper understanding of how to enhance Physics education. In providing a more holistic view of what drives the success of students in Physics, the purpose of this study is to determine the interrelationships as well as the direct and indirect effects of mathematical proficiency, scientific reasoning, and metacognitive skills on performance in Physics. Particularly, the study sought to answer the following questions:

i. What general model can describe the interrelationship of mathematical proficiency, scientific reasoning, and metacognition to students' performance in Physics?
ii. What variables directly affect the performance of students in Physics?
iii. What are the variables that indirectly affect the performance of students in Physics?

2. Literature Review

2.1. Mathematical Proficiency and Performance in Physics

According to Turner (2010), mathematical proficiency includes a range of skills necessary for applying mathematical knowledge in real-world situations. Kilpatrick et al. as cited by Brijlall & Ivasen (2022) clarified the competencies to include conceptual understanding, procedural fluency, strategic competence, adaptive thinking, and productive disposition, while Yulian and Wahyudin, (2018) noted that students' ability to solve problems is greatly impacted by their mathematical proficiency.

When it comes to Physics achievement, students with good mathematical ability had a higher mean percentage gain than those with low mathematical proficiency. The student's success in Physics (Electromagnetism) is influenced by a combination of mathematics proficiency, instructional tactics, and gender (Ibibo & Francis 2017). Moreover, a linear relationship between mathematics and physics achievement supports the premise that mathematical capacity is necessary for learning Physics efficiently (Chen et al., 2021; Long & Jiar, 2014). The higher the mathematical competency of the students, the better their scores on Physics tests as compared with the students with low mathematical competency (Doris, 2019). Most problems and concerns raised by students studying Physics came from the application of mathematical concepts and processes in the subject (Mumthas & Abdulla, 2019).

2.2 Scientific Reasoning and Physics Performance

Scientific reasoning is a rigorous method of problem-solving, critical thinking, and assessment of available evidence to explain results (Morris et al., 2012; Barz & Achimaş-Cadariu, 2016). Scientific reasoning also refutes biased and motivated reasoning, is dedicated to result-based evidence, and insists on impartiality (Cusimano & Lombrozo, 2021).

Students with a high-level reasoning ability perform at par for both the algorithmic and concept-based problems, while those with average and low reasoning abilities perform equally for the concept-based problems. The high reasoners outperform the average and low reasoners; thus, their reasoning ability relates to their problem-solving performance. Logically, the latter would indicate that possessing more formal reasoning patterns would mean that the students do better in constructing and applying the complex concepts of problem-solving (Fabby & Koenig, 2014). In support, there is a strong correlation between Lawson's Classroom Test of Scientific Reasoning (LCTSR) scores and normalised scores on concept inventories in Physics. The correlation is strongest for content that can be categorised as mostly theoretical, meaning a lack of directly observable exemplars, and weakest for content categorised as mostly descriptive, where directly observable exemplars are abundant (Moore & Rubbo, 2012). This can be attributed to the fact that the problem-solving method is one of the factors in developing scientific reasoning among students (Wuriyudani et al., 2018) using their existing knowledge on the subject (Hejnova et al., 2018). Moreover, in scientific reasoning tests, the result shows that students’ scientific reasoning is dominated by competence level; however, claims, arguments, and confidence of the students in answering the questions are still low. Also reported is false reasoning and weak conceptualisation when answering Physics problems (Erlina et al., 2018).

2.3 Metacognitive Skill and Physics Performance

According to Cihanoglu (2012), Rahimi and Katal (2012), Jaleel and Premachandran (2016), and Mondal (2023), metacognitive skills are the capacity to reflect on one's thinking and to be aware of one's mental processes; it is also referred to as metacognitive awareness, and it involves developing self-regulation and reflecting on one's prior knowledge. These abilities are essential for effective learning because they help students become more aware of what they do and comprehend how to apply their learning tactics in various contexts. Acquiring certain learning strategies is just as important as developing reflective learning skills to develop metacognitive abilities.

Developing metacognitive skills is essential for enhancing critical thinking and problem-solving abilities. It is important for a student since it aids in planning, monitoring, and assessing their abilities (Fauziah et al., 2022). Metacognitive skills and academic achievement are positively correlated (Mondal, 2023) as with Physics learning efficiency (Bogdanović et al., 2015).

Students who used metacognitive strategies in learning Physics gained higher scores than those using traditional strategies (Silitonga et al., 2020). In a study by Saaidin (2020) using SEM, students' academic performance in Physics was indirectly affected by metacognition with motivation and commitment as mediators. Meanwhile, Wider and Wider (2023) proved that monitoring and evaluation skills, the subcomponents of metacognition, are the main significant predictors of Physics problem-solving skills. As supported by Rahayu and Hertanti (2020), there is a positive association between metacognitive awareness and Physics problem-solving skills, and Ismiyati et al. (2019) found that using a problem-based learning approach in Physics instruction can enhance students' metacognitive skills. However, although there is a correlation between the student’s metacognition and problem-solving skills in Physics, students still have only a moderate level of metacognition (Anandaraj & Ramesh, 2011). Yanti et al. (2017) discovered that there is generally a need for improvement because students' metacognitive skills in answering abstract Physics issues are low.

Metacognitive skills are rarely used, as is the strategy on how to use them (Rahimi & Katal, 2012).

3. Methodology

3.1. Research Design

This research employed a quantitative design. Variables were defined, and correlations between and among the variables were performed through Structural Equation Modelling. The level of the students' proficiency in each variable and their academic performance was determined.

The interrelationship of the variables in the study was determined by employing a correlational approach. Specifically, the Structural Equation Modelling (SEM) method chosen facilitates the analysis of likely relationships among the selected variables which include mathematical proficiency, scientific reasoning, metacognitive skills, and academic performance in physics. The SEM provides a detailed analysis of the direct and indirect linkages between the variables.

3.2. Sampling Technique

The study involved the Grade 10 students who were currently enrolled for the School Year (S.Y.) 2022–2023 in public secondary schools in the Second District of the Schools Division of Ifugao. The sample size that was selected to become a member of each school was determined by the use of the Raosoft calculator. Table 1 shows the population and the computed sample size for each school for each municipality. Simple random sampling was used to enlist the participants for the study. The names of the students were obtained through the class adviser. The participants were selected by generating random numbers using Microsoft Excel.

Table 1: Population and sample size of the student respondents in the study

District	(N)	(n)	Percentage
Alfonso Lista District			
School A National High School	30	28	7.59%
School B National High School	58	51	13.82%
School C National High School	70	60	16.26%
Aguinaldo District			
School D National High School	22	21	5.69%
School E National High	17	17	4.61%
School F National High School	32	30	8.13%
School G Science High School	20	20	5.42%
Mayoyao District			
School H National High School	27	26	7.04%
School I National High School	33	31	8.40%
Banaue District			
School J National High School	26	25	6.78%
School K National High School	70	60	16.26%
TOTAL	405	369	100.00%

3.3. Research Instrument

The tool used for the study was divided into three parts: 1. Measurement of mathematical proficiency 2. Measurement of scientific reasoning 3. Measurement of metacognitive skill assessment.

The students began the examination by taking the Basic Skills Diagnostic Test, which was intended to assess the student's level of mathematical knowledge. Epstein (1997) designed the BSDT questions in the manner of the learner's perspective. The BSDT Test was evaluated across institutions and was subjected to the diversity in the demography of the students. More recently, the BSDT Test was used by Nye et al. (2018) as a far-transfer test for the likely universal improvements of mathematical skills. Similarly, Dame et al. (2019) have used this test to gauge students' learning in Mathematics because the test contains problems that critically engage the students in appraising fundamental mathematical matters and employing basic mathematical skills in daily quantitative situations.

The instrument for basic skill diagnostic testing by Epstein (1997)was pilot-tested with 47 Grade-10 non-participating students and had a Cronbach's alpha of 0.86, which indicated good internal consistency, as shown in Table 2. In other words, the items in Epstein's test are highly intercorrelated and measure the same underlying ability or construct; therefore, all items in the test were retained.

Table 2: Pilot testing of Basic Skill Diagnostic Test on Mathematical Proficiency

	Mean	Std. Deviation	Scale Mean if Item Deleted	Cronbach's Alpha if Item Deleted
Item 1	0.43	0.496	7.122	0.860
Item 2	0.62	0.487	6.935	0.855
Item 3	0.77	0.422	6.783	0.858
Item 4	0.41	0.492	7.144	0.858
Item 5	0.19	0.400	7.361	0.854
Item 6	0.16	0.377	7.389	0.856
Item 7	0.36	0.593	7.190	0.860
Item 8	0.24	0.435	7.310	0.850
Item 9	0.13	0.345	7.421	0.860
Item 10	0.41	0.493	7.139	0.863
Item 11	0.80	0.403	6.755	0.861
Item 12	0.11	0.317	7.446	0.861
Item 13	0.20	0.406	7.353	0.857
Item 14	0.06	0.244	7.495	0.857
Item 15	0.52	0.500	7.030	0.854
Item 16	0.33	0.478	7.217	0.861
Item 17	0.33	0.475	7.226	0.853
Item 18	0.30	0.464	7.253	0.852
Item 19	0.25	0.441	7.299	0.857
Item 20	0.06	0.244	7.495	0.857
Item 21	0.35	0.482	7.207	0.851
Item 22	0.23	0.428	7.321	0.850
Item 23	0.25	0.438	7.304	0.850

Item 24	0.06	0.232	7.495	0.862
Cronbach Alpha		**0.86**		

Additionally, to enhance the validation of the instruments, the researcher sought assistance from both the subject teachers and master teachers to check the alignment of the tests to the K-12 curriculum. Each item was categorised into the following Learning Contents:

Topics (Learning Content)	**Item number**
Number and Number Sense	1,2,3,4,5,9,10,
Geometry	6,12,13,
Patterns and Algebra	14, 15, 16, 17, 18,19,20,21,22,23
Statistics and Probability	24
Measurements	7, 8

The second half was the Lawson Classroom Test of Scientific Reasoning (LCTSR). The first ten pairs of questions in the LCTSR followed the standard two-tier pattern. In this construction, the first of the two questions required the respondents to determine the outcome of a certain concept while the other question was to be answered in such a way that the respondents' answer in the previous question was explained and rationalised. The last two questions (questions 23–24) were experimental outcomes to be disproved with given hypotheses.

This instrument was recently applied by Bao et al. (2018), who determined that the LCTSR demonstrates good reliability, with Cronbach's alpha values greater than 0.8 for the individual and pair scoring methods, particularly with the control of the test length. The findings of this study on reliability through the pilot testing of LCTSR showed that the instrument had a satisfactory level of internal consistency.

The results of the pilot testing are presented in Table 3. Cronbach's alpha for 47 respondents was 0.74, indicating that there was an acceptable internal consistency of the items; hence, all items were retained.

Table 3: Pilot testing result of the Lawson Classroom Test of Scientific Reasoning

	Mean	**Std. Deviation**	**Scale Mean if Item Deleted**	**Cronbach's Alpha if Item Deleted**
Item 1	0.605	0.4895	6.54	0.734
Item 2	0.450	0.4981	6.69	0.740
Item 3	0.316	0.4656	6.83	0.726
Item 4	0.300	0.4588	6.84	0.738
Item 5	0.215	0.4116	6.93	0.736
Item 6	0.199	0.3997	6.95	0.733
Item 7	0.199	0.3997	6.95	0.731
Item 8	0.188	0.3913	6.96	0.730

Item 9	0.191	0.3934	6.95	0.736
Item 10	0.136	0.3435	7.01	0.722
Item 11	0.283	0.4513	6.86	0.732
Item 12	0.226	0.4189	6.92	0.728
Item 13	0.354	0.4789	6.79	0.735
Item 14	0.316	0.4656	6.83	0.733
Item 15	0.172	0.3776	6.97	0.735
Item 16	0.297	0.4576	6.85	0.729
Item 17	0.218	0.4134	6.93	0.733
Item 18	0.147	0.3547	7.00	0.738
Item 19	0.559	0.4972	6.59	0.739
Item 20	0.335	0.4727	6.81	0.737
Item 21	0.199	0.3997	6.95	0.736
Item 22	0.441	0.4972	6.70	0.745
Item 23	0.324	0.4687	6.82	0.737
Item 24	0.474	0.5000	6.67	0.752
Cronbach Alpha		***0.74***		

The respondents' answers were checked using a pair-scoring technique with one point being awarded to each pair of questions. The scores of respondents were analysed using Piaget's system of formal operational, early and late transitional, and concrete operational reasoners, as shown below:

Table 4: Respondents' scores using Piaget's system

Range	**Description**	**Definition**
11 to 13	Formal operational reasoners	Formal operational reasoners possess the ability to think logically and systematically about abstract and hypothetical concepts. They can engage in deductive reasoning, manipulate symbols and variables, and generate hypotheses and theories.
8 to 10	Late transitional reasoners	The student is almost fully using the new, more advanced way of scientific thinking. They have mostly discarded the old way and are becoming skilled in new scientific reasoning.
5 to 7	Early transitional reasoners	Students display characteristics of both the previous and the upcoming cognitive stages. They are starting to use elements of more advanced scientific thinking, but they still rely on some of their old ways of thinking.

0 to 4	Concrete operational reasoners	Concrete operational reasoners are individuals who exhibit a stage of cognitive development characterised by logical thinking based on concrete objects and real-life situations. However, their thinking is limited to concrete, tangible experiences and they may struggle with abstract or hypothetical situations

The Metacognitive Awareness Inventory (MAI) instrument consists of indicators developed by Schraw and Moshman (1995) and validated by Omprakash et al. (2021) with an internal consistency of ≥0.9.

In a pilot study, the respondents completed the MAI to determine the internal consistency of the indicators. Unlike the typical binary response style which normally generates either disagreement or agreement, the questionnaire adopted a Likert scale that resulted in a diversity of rating of the responses. The MAI tool had participants rate the level of awareness of the statements.

A pilot test of 47 participants produced a Cronbach alpha of 0.95 showing that the items had a strong internal consistency, as indicated in Table 5, hence, all items were retained. The MAI items covered a wide spectrum of metacognitive awareness areas, like cognition, understanding, and the ability to influence thinking. This assessment consisted of 52 items measuring eight scales in which participants were asked to describe their thinking processes: declarative knowledge, procedural knowledge, conditional knowledge, planning, information management strategies, monitoring, debugging strategies, and evaluation of learning.

Table 5: Pilot testing result on Metacognitive Awareness Inventory

	Mean	Std. Deviation	Scale Mean if Item Deleted	Cronbach's Alpha if Item Deleted
Item 1	2.860	0.8916	139.27	0.954
Item 2	2.700	0.7438	139.43	0.954
Item 3	2.807	0.8989	139.32	0.953
Item 4	2.689	0.8321	139.44	0.954
Item 5	3.053	1.8191	139.07	0.957
Item 6	2.941	0.8575	139.18	0.954
Item 7	2.613	0.8751	139.51	0.954
Item 8	2.686	0.9101	139.44	0.954
Item 9	2.815	0.9053	139.31	0.954
Item 10	2.913	0.8681	139.21	0.954
Item 11	2.692	0.8996	139.43	0.953
Item 12	2.443	1.0140	139.68	0.954
Item 13	2.745	0.8929	139.38	0.953
Item 14	2.706	0.8613	139.42	0.953
Item 15	2.922	0.8991	139.20	0.954

Item 16	2.496	0.9045	139.63	0.954
Item 17	2.487	1.0266	139.64	0.954
Item 18	2.854	0.8875	139.27	0.953
Item 19	2.843	0.8760	139.28	0.953
Item 20	2.625	0.8308	139.50	0.953
Item 21	2.714	0.9404	139.41	0.953
Item 22	2.695	0.8990	139.43	0.953
Item 23	2.877	0.8718	139.25	0.953
Item 24	2.594	0.9181	139.53	0.953
Item 25	3.014	0.9040	139.11	0.954
Item 26	2.885	0.8681	139.24	0.954
Item 27	2.725	0.8791	139.40	0.953
Item 28	2.683	0.8129	139.44	0.953
Item 29	2.714	0.8723	139.41	0.953
Item 30	2.762	0.8426	139.36	0.953
Item 31	2.616	0.9247	139.51	0.954
Item 32	2.549	0.9337	139.58	0.953
Item 33	2.636	0.9098	139.49	0.953
Item 34	2.560	0.9025	139.57	0.953
Item 35	2.669	0.8693	139.46	0.953
Item 36	2.725	0.9438	139.40	0.953
Item 37	2.415	0.9695	139.71	0.954
Item 38	2.630	0.8824	139.50	0.953
Item 39	2.695	0.8769	139.43	0.953
Item 40	2.734	1.4139	139.39	0.954
Item 41	2.667	0.8369	139.46	0.953
Item 42	2.958	0.9186	139.17	0.954
Item 43	2.790	0.8434	139.34	0.953
Item 44	2.602	0.8924	139.52	0.953
Item 45	2.768	0.9175	139.36	0.953
Item 46	3.090	0.9258	139.04	0.954
Item 47	2.496	0.8888	139.63	0.954
Item 48	2.658	0.8352	139.47	0.953
Item 49	2.810	0.8530	139.32	0.953
Item 50	2.801	0.8428	139.32	0.953
Item 51	2.739	0.9522	139.39	0.954
Item 52	2.964	0.9399	139.16	0.954
Cronbach Alpha		***0.95***		

Participants were asked to rate their level of awareness for each statement on the MAI, allowing for a more detailed analysis of their metacognitive processes and enhancing the instrument's sensitivity to variations in respondents' perceptions. The respondents evaluated their metacognitive skills using the scale below:

Table 6: Respondents' evaluation of their metacognitive skills

Scale	Description	Definition
4	Very highly aware	This level indicates an exceptional degree of 100% metacognitive awareness. Individuals who are very highly aware possess a deep understanding of their cognitive processes, learning strategies, strengths, and weaknesses.

3	Highly aware	Being highly aware in the Metacognitive Inventory refers to a significant level of 67% metacognitive awareness. Individuals at this level have a good understanding of their cognitive processes and can recognise their thinking patterns and strategies.
2	Slightly aware	Slightly aware individuals have a minimal level of 33% metacognitive awareness. They have a limited understanding of their cognitive processes, learning strategies, and the impact of their thinking on their learning outcomes. They may lack self-reflection skills and have little awareness of their strengths and weaknesses.
1	Not aware	The individual is not aware of their own cognitive process or the learning strategies that may impact their thinking.

3.4. Ethical Consideration

Ethical considerations were taken into account to safeguard the participants' privacy and interests. A request letter was sent to the Superintendent of Schools, requesting permission to carry out the research at the specified schools. Participants were fully informed of the research aims before the start of the study and the study's goals were explained to prevent misconceptions. The participants were assured of anonymity and that personal information would be restricted to those involved in the research. All participants signed data privacy and informed consent forms.

3.5. Data Gathering Procedure

Data-gathering took place from January to June 2023 to cover the second grading period of the students in which Physics topics were discussed. The researcher personally administered the instruments to the students and read out the instructions. Once the researchers felt that the students understood the instructions, the students were asked to answer Part I of the survey form. The researcher read the guidelines twice for Parts II and III to ensure that students understood them clearly. Each test duration was set for 1 hour and 30 minutes for the mathematical proficiency test and the scientific reasoning test using Lawson's Classroom Scientific Reasoning exam (Parts II and III). The Metacognition Awareness Inventory was taken in Part IV without a time limit to ensure the students understood the purpose and value of this section.

Academic achievement was defined as the inclusion of accomplished tasks (Abba, 2018) and the attainment of educational objectives through activities and discourses (Paudel, 2021) often measured through numerical grades through various assessments or examinations (Narula & Sindhwani, 2016; Ariza et al., 2018); hence, the researcher considered academic performance as the student’s academic grades. The student’s academic performance was the result of various formative and summative assessments targeting the same competencies of the K-

12 curriculum under Physics, known in the curriculum as Force, Motion, and Energy across all public schools in the Philippines. The students' Physics academic achievement was obtained through collaboration with the subject teacher, with the full approval of both the students and the school head. The subject teacher gave the students' grades, which were written on the lower left corner of their sheets.

The interrelationships between and among the variables were intended to be represented by the Structural Equation Model (SEM) using AMOS software. The model helped in observing patterns and relationships among the variables, giving a more detailed understanding of how each factor affected or was affected by others in the study. After tallying and reviewing the obtained results, the questionnaires underwent a careful packing and sealing procedure.

4. Results and Discussion

4.1 General Model Showing the Interrelationship of Mathematics Proficiency, Scientific Reasoning Skills, Metacognitive Skills, and Academic Performance in Physics

Figure 1 shows the established model for mathematics proficiency, scientific reasoning skills, metacognitive skills, and academic performance in Physics. As confirmed in Table 7, the CMIN/DF value of 2.359 together with a P-value of 0.068 confirmed that Figure 1 is the model that best describes the actual conditions of the variables. Overall, the model is accepted as determined by TLI, GFI, AGFI, CFI, and RMSEA as indicated by the "good" indicator.

Table 7: Summary of the Goodness Fit Index of the model

Goodness of Fit Index	Acceptable Value	Model Value	Indicator
X^2- Chi-Square	Smaller Value	2.36	Good
Significance Probability	≥0.05	0.068	Good
CMIN/DF	≤2.00	2.359	Good
GFI	≥0.90	0.934	Good
AGFI	≥0.90	0.916	Good
CFI	≥0.95	0.985	Good
RMSEA	≥0.08	0.061	Good
Tucker-Lewis Index	≥0.95	1.000	Good

Based on the accepted general model in Figure 1, mathematics proficiency was directly related to academic performance in Physics; three dimensions of the scientific reasoning skills, particularly formal operational, late transition, and early transition had a direct effect on the academic performance of students in Physics. In addition, the metacognitive skills of declarative, evaluation, and debugging had a direct effect on academic performance.

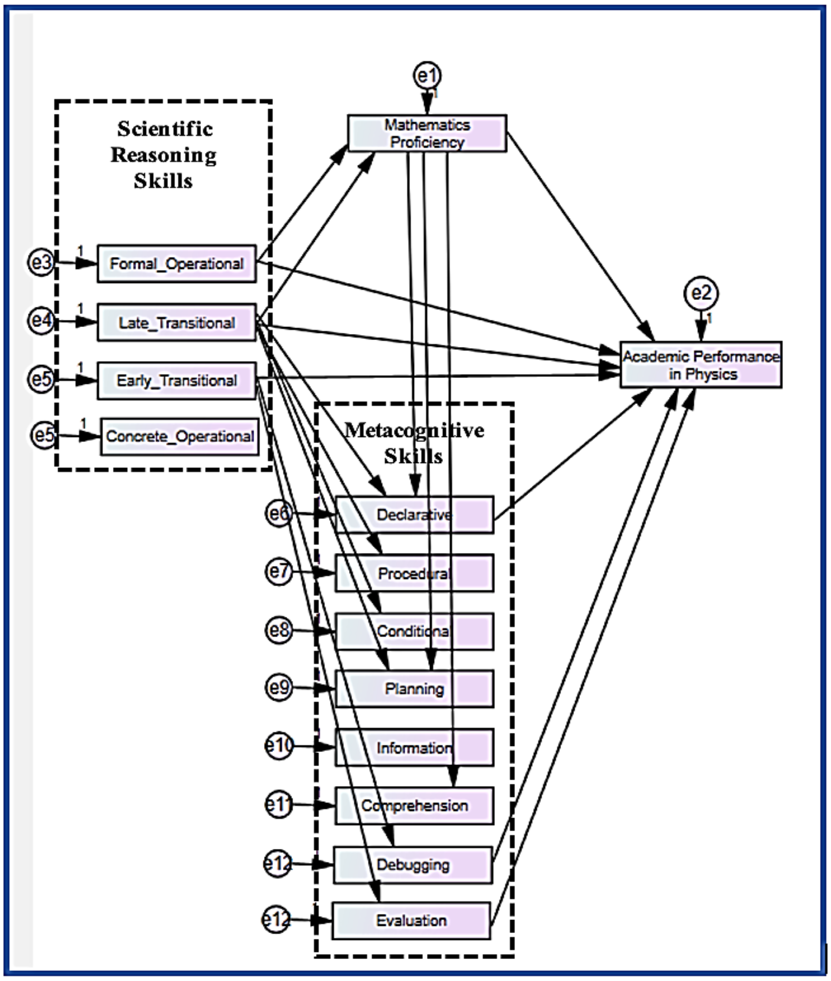

Figure 1: Generally Accepted Model Showing the Interrelationship of Mathematics Proficiency, Scientific Reasoning Skills, Metacognitive Skills, and Academic Performance in Physics

Moreover, mathematics proficiency influenced the domain of metacognitive skills such as declarative, planning, and comprehension. In the region of scientific reasoning skills, the late transitional level influenced the domain of the metacognitive skills (declarative and debugging) and mathematics proficiency; scientific reasoning skills - early transitional have an effect on debugging and evaluation while scientific reasoning skills- Formal Operational directly affects mathematics proficiency.

According to Utami et al. (2023), promoting metacognitive skills enhances the problem-solving abilities that can be used in Mathematics and Physics. This finding is further supported by Djudin (2020), who found that metacognitive practice with mathematical knowledge improved students' Physics problem-solving. Furthermore, Cihlár et al. (2020) suggest that scientific reasoning, along with its sub-categories, such as proportional reasoning, control of variables, and probability reasoning is associated with the student's mathematical proficiency and the ability to utilise existing knowledge in their academic subjects. Additionally, Villena and Caballes (2019) highlight the positive and moderate relationship between reasoning skills and metacognitive skills, with scientific reasoning demonstrating an impact on academic excellence, while Rickey and Stacy (2000) believe that metacognition refines student ideas about scientific ideas.

4.2. Direct Effect of Mathematics Proficiency, Scientific Reasoning, and Metacognitive Skills to Academic Performance in Physics

Figure 2 shows that mathematics proficiency, scientific reasoning skills (formal operational, late transitional, early transitional), and metacognitive skills (declarative, debugging, and evaluation) had a direct effect on academic performance in Physics.

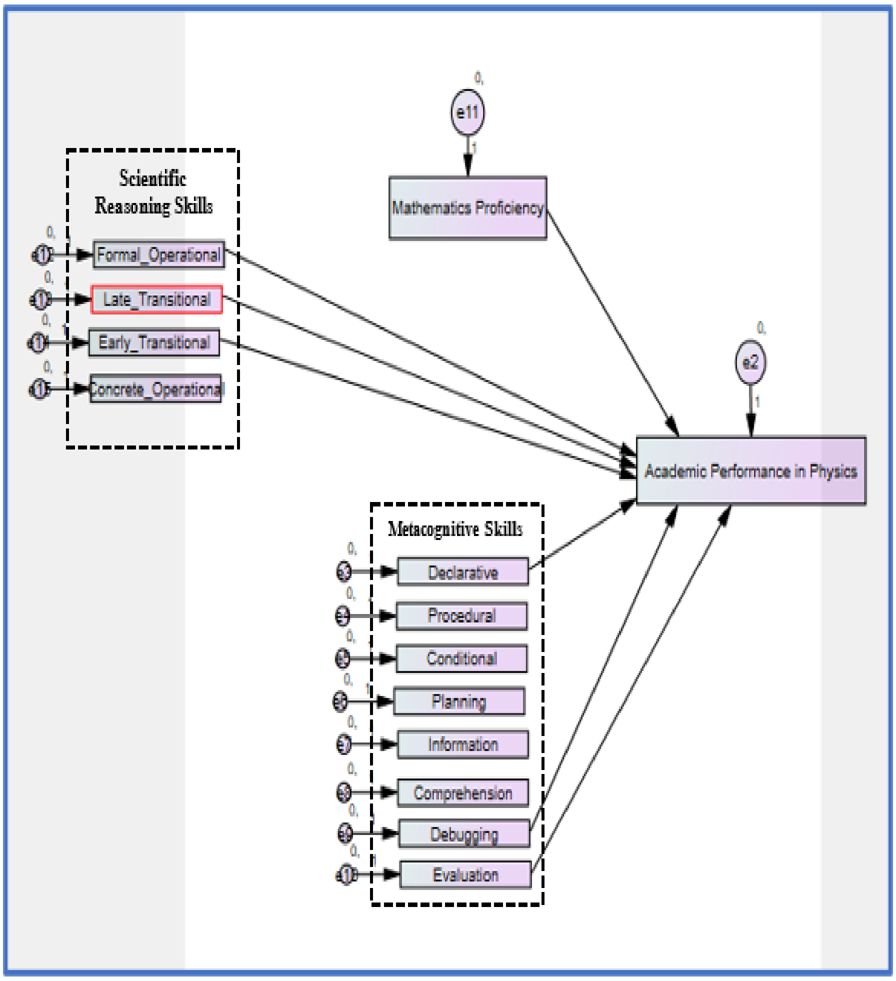

Figure 2: A portion of the Accepted Model reflects the direct paths from Mathematics Proficiency, Scientific Reasoning Skills, and Metacognitive Skills to Academic Performance in Physics

Table 8 summarises the direct effects of the predictors on academic performance in Physics. Mathematics proficiency has the greatest influence on academic performance (E=4.150, p<0.05) followed by metacognitive skill: declarative (E=1.17, p<0.05), scientific reasoning: formal operational (E=.19, p<0.05), late transitional (E=0.352, p<0.05), early transitional (E=0.321, p<0.05), while metacognitive skill: debugging (E= 0.30, p<0.05) and evaluation (E= 0.16,1 p<0.05) has the least effect.

Table 8. Direct effects of predictors on academic performance in Physics

			Estimate	C. R.	p-value
Academic Performance	←	Mathematics Proficiency	4.150	2.581	0.01
Academic Performance	←	Formal Operational	0.910	2.632	0.00
Academic Performance	←	Late Transitional	0.352	1.835	0.00
Academic Performance	←	Early Transitional	0.321	2.632	0.04
Academic Performance	←	Declarative	1.17	2.388	0.03
Academic Performance	←	Debugging	0.30	2.55	0.02
Academic Performance	←	Evaluation	0.161	1.629	0.00

Figure 2 and Table 8 provide evidence that mathematics proficiency had the greatest direct effect on the academic performance of the students, indicating that the Grade 10 participants have a solid mathematical foundation and can understand essential algebra concepts. The students' Physics performance was improved owing to their mathematical skills; students who grasp basic algebraic expressions and equations can solve physics issues appropriately. The student's capacity to perform arithmetic in fractions, decimals, and percentages allowed them to provide accurate answers in Physics. Furthermore, the students can solve problems in Physics with confidence when they connect the learned mathematical concepts to their Physics lessons.

Students who understood the important algebraic principles demonstrated more effective problem-solving skills in Physics which led to increased academic performance; therefore, the direct effect of Mathematics on academic achievement in Physics demonstrates that concepts from Mathematics are also used in Physics, as the two fields are interwoven (Galili, 2018). Moreover, mathematical proficiency influences students' Physics performance, supporting the importance of Mathematics in studying Physics (Chen et al., 2021; Tashpulatovich & Qizi, 2021). Doran (2017) revealed that quantification and derivation principles in Mathematics help in Physics learning by connecting acquired concepts to actual-life situations.

In scientific reasoning skills, only formal operational, late transitional, and early transitional reasoners had a direct impact on the academic performance of the Grade 10 Physics students, indicating that students used critical thinking to identify problems. Scientific reasoning skills enable them to analyse information, recognise patterns, and draw conclusions. In addition, they were able to generate hypotheses based on observation and to construct experiments to test them. Students capable of scientific reasoning could systematically approach problems, collect data, and apply appropriate concepts. Such reasoning implied building models to represent systems in Physics. Meanwhile, students with strong scientific reasoning skills might approach problems methodically, find pertinent data, and apply the right principles. The higher reasoning skills of students enable them to excel in problem-solving and algorithms in Physics (Fabby & Koenig, 2014). Formal, logical, cognitive thinking is particularly required in studying Physics (Saayman, 1991), and students who exhibited critical thinking, hypothesis formulation, and experimental design skills demonstrated higher levels of academic achievement.

Only declarative knowledge, debugging strategies, and evaluation aspects of metacognitive skills were directly related to academic success in Physics. The students' declarative knowledge means that they could arrange their study sessions, assign time to the various topics, and set goals. Also, students could learn by talking and showing, which motivates them to analyse and understand concepts in Physics, as well as check their understanding of the Physics lesson. Apart from this, students could notice when a topic was difficult and adjust their study techniques accordingly. The metacognitive skill of debugging techniques helped Grade 10 students identify where their understanding of Physics topics was wrong or incomplete, enabling them to evaluate their mistakes and flaws in problem-solving and locate the root cause of problems to rectify the wrong concepts. They were able to change their problem-solving strategies in response to their mistakes to avoid repeating the same errors. Lastly, of the metacognitive skills, evaluation had a direct, significant correlation with academic achievement in Physics, supporting the evidence that students evaluate their comprehension, re-examine their methods, and learn from their mistakes. Students could also measure how well they scored on a certain evaluation and judge if their learning strategies were effective.

In a study by Mondal (2023), metacognitive skill as determined by the student's metacognitive awareness is associated with their scholastic and academic achievement. In support, metacognition is found to be one of the predictors of academic performance (Pucillo & Pérez, 2023). In the context of Physics, Saaidin (2020) found that the practice of metacognition influences motivation and commitment, which in turn, affects academic performance. Similarly, Bogdanović et al. (2015) reveal a positive correlation between metacognitive skills and academic performance in Physics; however, at a deeper level, students with metacognitive skills, particularly in declarative knowledge, planning skills, and evaluation processes showed greater capacity to monitor, regulate, and change their learning strategies, resulting in higher academic achievement. Content knowledge in Physics helps students overcome obstacles by deriving solutions to ill-structured problems (Milbourne & Wiebe, 2017) while reflection on the concepts gives students a clearer understanding of the Physics topics (Sarwar & Trumpower, 2015). These findings evidence the importance of developing metacognitive skills for effective Physics learning and problem-solving.

4.3. Indirect Effect of Mathematics Proficiency, Scientific Reasoning, and Metacognitive Skills to the Academic Performance in Physics

Table 9 summarises the corresponding indirect effects of each predictor on academic performance in Physics. As illustrated in Figures 3, 4, and 5, the table illustrates the estimates and direction of each predictor and academic performance when mediated through other variables.

Table 9: The indirect effects of variables on academic performance in Physics

					Estimate	C. R.	p-value
Academic Performance	←	Formal Operational	←	Mathematics proficiency	0.872	1.52	0.03
Academic Performance	←	Declarative	←	Mathematics proficiency	0.821	1.11	0.02
Academic Performance	←	Mathematics Proficiency	←	Formal Operational	1.33	1.28	0.00
Academic Performance	←	Mathematics Proficiency	←	Late Transitional	0.882	0.82	0.03
Academic Performance	←	Early Transitional	←	Evaluation	0.617	2.197	0.02
Academic Performance	←	Mathematics Proficiency	←	Planning	1.73	1.32	0.00
Academic Performance	←	Early Transitional	←	Planning	1.52	1.23	0.01
Academic Performance	←	Late Transitional	←	Declarative	0.937	0.95	0.03
Academic Performance	←	Mathematics Proficiency	←	Declarative	0.921	1.38	0.00

Scientific reasoning – formal operational and metacognitive skill – declarative had a positive indirect effect on academic performance in Physics through mathematics proficiency with estimates of 0.872, $p<0.05$ and 0.821, $p<0.05$, respectively. Furthermore, mathematics proficiency indirectly influenced academic performance through scientific reasoning: formal operational (E= 1.33, $p <0.05$) and late transitional (E=0.882, $p<0.05$); and with metacognitive skills: planning (E=1.73, $p<0.05$) and declarative (E=.921, $p<0.05$). Scientific reasoning skills: early transitional, indirectly affected academic performance through evaluation (E=.617, $p<0.05$) and planning (E=1.52, $p<0.05$) while scientific reasoning: late transitional, had an indirect influence on academic performance via metacognitive skills: declarative (E=0.937, $p<0.05$).

Based on Table 9, Figure 3 presents the section of the accepted model depicting the indirect connections of Mathematics Proficiency on metacognitive skills and scientific reasoning skills toward Academic Performance in Physics. This means that metacognitive skills: declarative, and scientific reasoning skills: formal operational tend to influence the academic performance of the students with the mediation of mathematics proficiency.

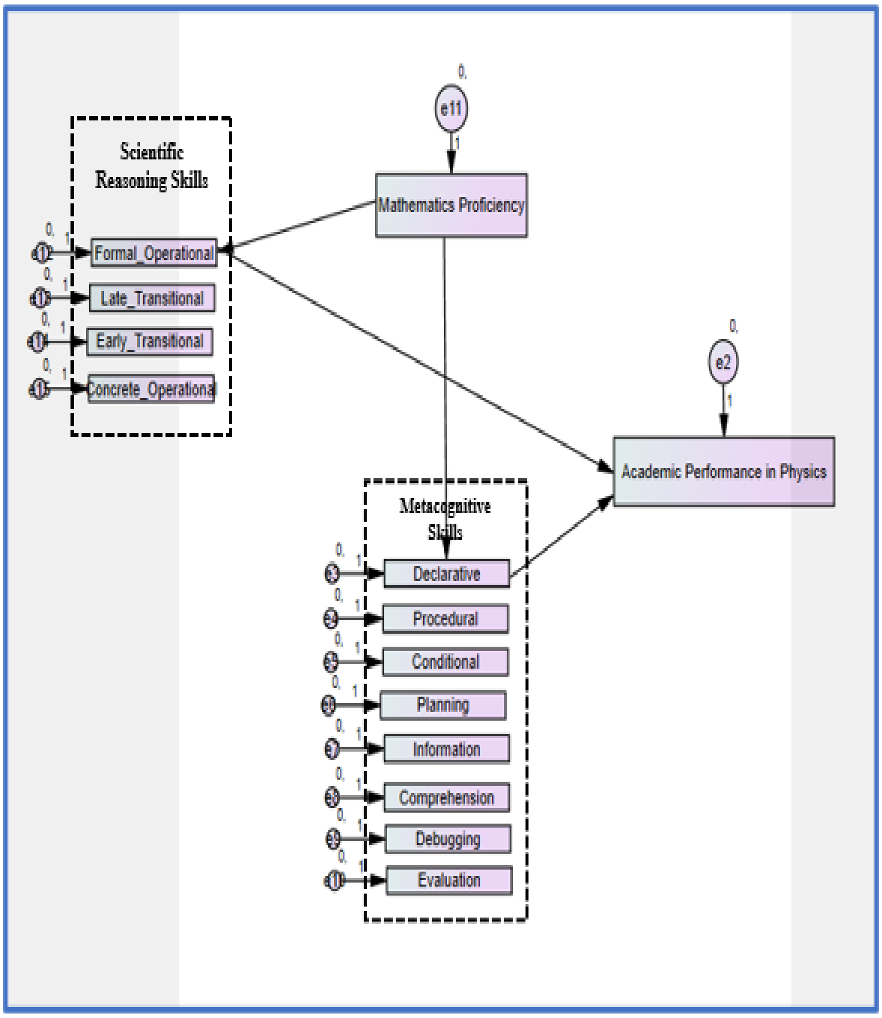

Figure 3: A portion of the Accepted Model reflecting the Indirect Effects of Mathematics Proficiency on Scientific Reasoning Skills and Metacognitive Skills towards Academic Performance in Physics

Thus, mathematical proficiency supplements scientific reasoning in which formal operational reasoners use abstract thinking, hypothesis formation, critical analysis, and problem-solving skills very closely related to mathematical proficiency. The students' scientific thinking abilities enabled them to approach Physics problems with greater comprehension and more effective problem-solving strategies. The arithmetic achievement of the students also has a positive relationship with the concrete operational stage of the learner (Lemoyne & Favreau, 1981) which proved that good reasoning ability enables students to solve and explain mathematical problems (Wulandari & Wutsqa, 2019). Furthermore, evidence showed that students' mathematical competency was one of the elements in their metacognitive skills, which led to improved academic achievement in Physics. Mathematically gifted students were more prone to use analytical thinking in their study methods, problem-solving strategies, and self-evaluation procedures. Students eventually gained this metacognitive capacity, declarative, as they applied analytical thinking, strategic planning, logical reasoning, and error recognition in their Mathematics courses. When metacognitive skills were applied in Physics learning, students performed better in terms of study plan group, understanding assessments, error identification, and technique adjustment. Metacognitive skill has a significant relationship with mathematical self-efficacy in solving Mathematics problems (Susilo & Retnawati, 2018; Izzati & Mahmudi, 2018). Also, metacognitive skill is the product of the evident mathematical abilities of the students in solving a problem (Pathuddin & Bennu, 2021) where metacognitive skills such as planning are important in solving Mathematics problems (Noor, 2022). Planning, particularly, accounts for the distinct different aspects of Mathematics problem-solving, in addition to influencing the functioning of non-verbal cognitive ability and working memory (Cai, 2015). In the context of this study, students who improved their metacognitive skills: declarative performed better academically in Physics.

Figure 4 shows the portion of the accepted model reflecting the indirect relationships between scientific reasoning skills to Mathematics proficiency and metacognitive skills toward academic performance in Physics as based on Table 9. The path implies that the scientific reasoning skills of students: formal and early transitional, greatly affected the mathematical proficiency of students in achieving better academic performance in Physics. This suggests that the development of scientific reasoning skills of Grade 10 students, particularly during the formal and early transitional stages of cognitive development, could positively influence students' mathematical proficiency and subsequently contribute to better academic performance in Physics.

Cihlár et al. (2020) proved that scientific reasoning is correlated with mathematical proficiency, and Tajudin and Chinnappan (2015) revealed that scientific reasoning plays an important role in problem-solving in Mathematics while Singley and Bunge (2014) show a positive relationship between scientific reasoning and mathematical proficiency, which is evident in childhood. This indicates that enhancing scientific reasoning is important to achieve mathematics proficiency and better academic performance in Physics since formal reasoners have been shown to achieve higher academic performance than transitional reasoners (Rohaeti et al., 2019).

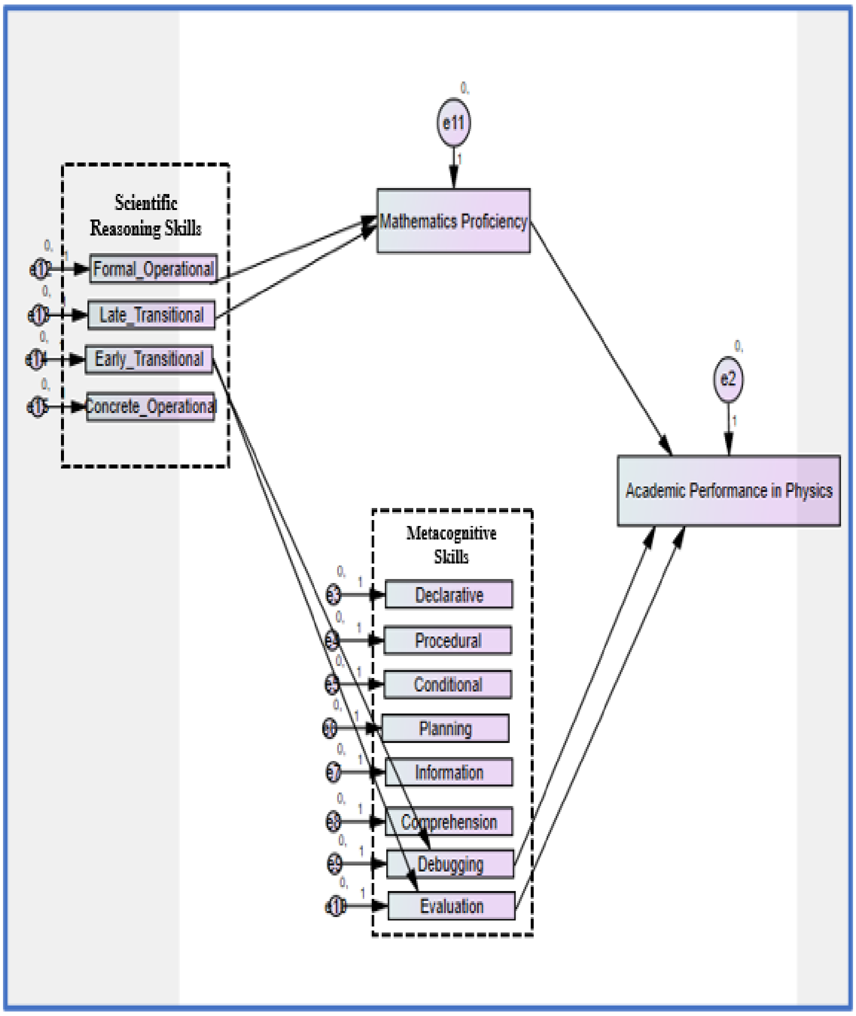

Figure 4: A portion of the Accepted Model reflecting the Indirect Effects of Scientific Reasoning Skills on Mathematics Proficiency and Metacognitive Skills toward Academic Performance in Physics

On the other hand, the path implies that the scientific reasoning skills of students: early transitional, greatly affect the metacognitive skills of students in the areas of debugging and evaluation to perform better academically in Physics. This suggests that the participants recognised and addressed errors, misconceptions, or gaps in their thinking or problem-solving processes and these skills were developed during transitional stage reasoning aided by effective debugging techniques. Further, students who critically analysed information reflected on problem-solving strategies and assessed the validity of their work. The emerging hypothesis testing and analytical thinking in the early transitional stage aligned with evaluation metacognition; thus, students with early stages of cognitive transition could increase metacognitive abilities such as debugging and evaluation, resulting in improved academic performance in Physics.

Based on Table 9, Figure 5 shows the portion of the accepted model reflecting the indirect effects of metacognitive skills on scientific reasoning skills and mathematics proficiency toward academic performance in Physics. The path implies metacognitive skills of students: declarative directly affected the scientific reasoning skills of students: early and late transition to attain better mathematical proficiency of Grade 10 students towards high academic performance in Physics. This suggests that those students who were informed of their cognitive processes and strategies were more likely to recognise their thought patterns, areas of strength, and areas that needed development. These students continued to improve their capacity for abstract thought and problem-solving. Metacognitive skills, like declarative, encouraged students to consider their understanding, identify misunderstandings, and fill in any gaps. A deeper knowledge of mathematical proficiency was important to achieve high academic performance in Physics. Moreover, the path implies the skills of students: planning and evaluation were one of the factors on scientific reasoning skills of students: early and late transition to attain better mathematical proficiency of Grade 10 students towards high academic performance in Physics. Those students who set goals, organised their approaches, allocated resources efficiently, critically analysed their work, reflected on their understanding, and assessed the quality of their solutions were those students who developed abstract thinking, systematic analysis, and hypothesis-testing abilities which led to better performance in Physics.

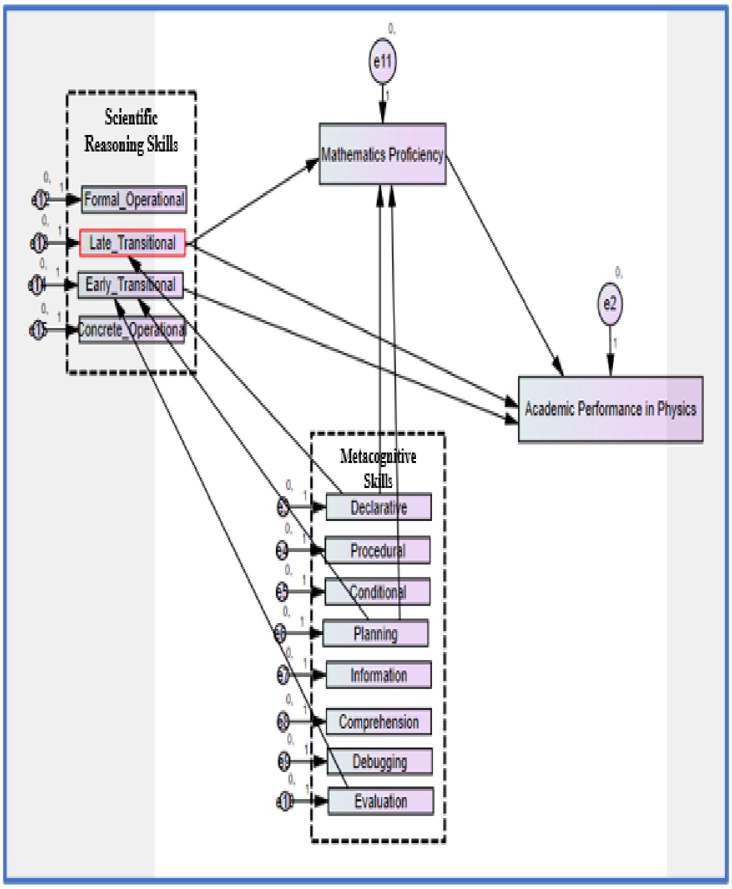

Figure 5. A portion of the Accepted Model reflecting the Indirect Effects of Metacognitive Skills on Scientific Reasoning Skills and Mathematics Proficiency toward Academic Performance in Physics

Metacognition and reasoning, also known as meta-reasoning, comprise self–management strategies that are linked together (Fletcher & Carruthers, 2012) since metacognition enhances reasoning through differentiated pedagogies (Kosior et al., 2019). There is also a moderate relationship between scientific reasoning skills to mathematics performance which indicates that scientific reasoning assists in mathematical problem-solving (Tajudin & Chinnappan, 2015). In the context of this study, variables such as mathematics proficiency enhanced scientific reasoning and metacognitive skills, while the development of scientific reasoning and metacognitive skills positively influenced each other, resulting in a mutually beneficial impact on student achievement in learning Physics.

5. Conclusions, Implications, and Recommendations

This study provided new insights into the relationship between mathematical proficiency, scientific reasoning skills, metacognitive skills, and their impact on academic achievement in Physics among Grade 10 students. The key findings emphasized the direct impact of mathematics proficiency, particular aspects of scientific reasoning skills (formal operational, late transitional, and early transitional), and metacognitive skills (declarative, debugging, and evaluation) on student achievement in Physics. These findings indicate the significance of developing a comprehensive skill set that includes mathematical proficiency, scientific reasoning abilities, and metacognitive capabilities to improve students' academic achievement in physics.

This research implies that effective teaching and learning in Physics requires heightened integration of mathematical competence, scientific reasoning ability, and metacognitive skills within the teaching-learning process. Hence, Physics curriculum planners need to consider the level of mathematical proficiency, scientific reasoning skills, and metacognitive skills of the students in crafting the Physics curriculum. Also, educators can develop interventions where these competencies are enhanced; teachers can provide activities that are aimed at enhancing scientific reasoning skills, integrating tasks to build metacognitive abilities, and incorporating mathematical concepts into Physics instruction such as game-based instruction. Lastly, the students need to learn strategies that can enhance their mathematical proficiency, metacognitive skills, and scientific reasoning skills as these can help them attain better Physics academic performance. Students need to connect mathematical principles in learning physics, analyse information, patterns, and data to further understand Physics, and reflect on their learning process to gain more effective strategies in learning Physics.

6. Scope and Limitations

This study focused on the interrelationship of mathematical proficiency, scientific reasoning skills, metacognitive skills, and academic performance in Physics of the Grade 10 students enrolled in public schools in District 2 of Ifugao Province, Philippines for the School Year 2022–2023. However, the study acknowledges certain limitations. Firstly, the scope was limited to Grade 10 students in a certain geographical area, restricting the generalisability of the findings in a broader context. Secondly, the analysis focuses only on mathematical proficiency, scientific reasoning, and metacognitive skills although other factors can potentially affect students' academic performance in Physics. Further, the academic performance of the students in Physics was based on their academic grades in Physics taken during the second grading period of the Grade 10 students with diverse subject teachers from different schools. Although each school followed the same learning competency in Physics, other factors can affect their academic performance. Lastly, other demographics and external factors are not directly examined, despite their potential influence.

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